

## RGM COLLEGE OF ENGINEERING &amp; TECHNOLOGY (AUTONOMOUS)

5th October 2021

I B.Tech II Semester (R20) End Examinations (Regular)

ADVANCED CALCULUS AND TRANSFORMATION TECHNIQUES

ECE

Time: 3 Hrs

Total Marks: 70

Note 1: Answer Question No.1 (Compulsory) and 4 from the remaining

2: All Questions Carry Equal Marks

- 1a Define Fourier Sine and its inverse Fourier Sine transform.
- b Find the Fourier series formula with Fourier coefficients for  $f(x)$  in  $(-\pi, \pi)$ ?
- c Find  $c$  value from Rolle's theorem for  $f(x) = x^2 - 3x + 2$  on  $(1, 2)$ .
- d If  $\beta(n, 2) = \frac{1}{42}$  then find the value of  $n$
- e Using triple integral find the volume of the parallelepiped rectangle bounded by  $x = y = z = 0$  and  $x = 1, y = 2$  and  $z = 3$ .
- f Show that  $Z(a^n) = \frac{z}{z-a}$  from definition of Z-transform.
- g Write the Change of scale property in Fourier transform.
- 2 a) Evaluate  $\int_0^1 x^3 \log\left(\frac{1}{x}\right)^4 dx$  (7)
- b) Evaluate  $\int_0^1 x^3 (1 - \sqrt{x})^5 dx$  (7)
- 3 Find the Fourier series to represent  $f(x) = x$  in the interval  $(0, 2\pi)$ ? (14)
- 4 a) Find the maximum and minimum distances of the point  $(3, 4, 12)$  from the sphere  $x^2 + y^2 + z^2 = 4$ . (7)
- b) Using Maclaurin's series expansion prove that (7)
- $$\log(1 + \sin x) = x - \frac{x^2}{2} - \frac{x^3}{6} - \frac{x^4}{384} + \dots$$
- 5 a) Calculate  $\iint r^3 dr d\theta$  over the area included between the circles  $r = 2\cos\theta$  and  $r = 4\cos\theta$  (7)
- b) By changing into polar coordinate evaluate  $\iint \frac{x^2 y^2}{x^2 + y^2} dx dy$  over the annular region between the circles  $x^2 + y^2 = a^2$  and  $x^2 + y^2 = b^2$  ( $b > a$ ) (7)
- 6 Find the Fourier Sine and Cosine transform of  $f(x) = \frac{e^{-ax}}{x}$  and deduce that (14)
- $$\int_0^{\infty} \frac{e^{-ax} - e^{-bx}}{x} \sin(sx) dx = \tan^{-1}\left(\frac{s}{a}\right) - \tan^{-1}\left(\frac{s}{b}\right).$$
- 7 State and prove initial and final value theorems. (14)