(ESTD-1995)

Rajeev Gandhi Memorial College of Engineering & Technology (Autonomous)

Nandyal-518 501, Kurnool (Dt.), Andhra Pradesh (State), INDIA

COURSE FILES

(Mathematics)

- CO's Mapping and its attainment
- Semester Session Plan
- Sample T-L Resources
- Assignments
- MID Term Papers and End Term Papers

RAJEEV GANDHI MEMORIAL COLLEGE OF ENGG.&

TECHNOLOGY, NANDYAL

(AUTONOMOUS)

(ESTD-1995)

Course File

Linear Algebra and Advanced Calculus

(LA&AC)

DEPARTMENT OF MATHEMATICS

Course File

Subject: **Linear Algebra and Advanced Calculus (LA&AC)**

INDEX

Dr. K. V. Suryanarayana Rao
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Deparment of Mathamatics
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Syllabus

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RAJEEV GANDHI MEMORIAL COLLEGE OF ENGG & TECHNOLOGY, NANDYAL
(AUTONOMOUS)
B. Tech. – I Year – I Sem. (2020-21)
For Branches: C.E., E.E.E., M.E., C.S.E., C.S.E (DS), C.S.E (BS).

	Linear Algebra and Advanced Calculus	BSC	3–0–0	3Credits

Course Objectives:

i) To familiarize the concepts of matrices and mean value theorems and their applications in engineering.

ii) To equip the students to solve various application problems in engineering through evaluation of Gamma, Beta functions and multiple integrals etc.,

Course Outcomes: After completion of the course the student will be able to:

CO1	Understand the use of matrices and linear system of equations in solving Network analysis, encoding and decoding in Cryptography and Quantum mechanics problems.
CO2	Apply the concept of Gamma and Beta functions in digital signal processing, discrete Fourier transform, digital filters and Oscillatory theory in engineering.
CO3	Analyze differential and integral calculus to solve improper integrals and its applications in many engineering disciplines.
CO4	Determine the process to evaluate double and triple integrals and understand its usage to find surface area and volumes of various bodies in engineering.
CO5	Identify the applications of advanced calculus & Linear algebra in electro-magnetic theory and in telecommunication engineering.

Mapping of Course outcomes with Program outcomes

CO	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO1	3	2	2	2	2	-	-	-	-	-	-	-
CO2	3	2	2	2	3	-	-	-	-	-	-	-
CO3	2	2	2	2	3	-	-	-	-	-	-	-
CO4	3	2	3	3	2	-	-	-	-	-	-	-
CO5	2	3	2	2	2	-	-	-	-	-	-	-

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UNIT – I

Matrices: Elementary row transformations – Rank – Echelon form, Normal form – Solutions of Linear System of Homogenous and Non Homogeneous equations.

UNIT – II

Eigen Values, Eigen vectors – Properties – Cayley – Hamilton Theorem – Inverse and Power of a matrix by Cayley – Hamilton theorem.

UNIT – III

Quadratic forms: Linear Transformation – Reduction of quadratic form to canonical form and their nature (Rank, Signature and Index).

UNIT – IV

Mean value theorems: Rolle's Theorem – Lagrange's Mean Value Theorem – (excluding proof). Simple examples of Taylor's and Maclaurin's Series.

Functions of several variables – Jacobian – Maxima and Minima of functions of two variables - Lagrange method of Multipliers with three variables only.

UNIT – V

Multiple integrals: – Evaluation of Double integrals (Cartesian and Polar) – Change of Variables – Change of order of Integration – Changing into Polar coordinates – Evaluation of triple integrals.

UNIT – VI

Special functions: Gamma function – Properties – Beta function – properties – Relation between Gamma and Beta functions – Evaluation of Integrals using Gamma & Beta functions.

Textbooks:

(i) B. S. Grewal, Higher Engineering Mathematics, Khanna Publications.
(ii) R. K. Jain, S. R .K. Iyengar, Advanced Engineering Mathematics, Alpha Science.
(iii) T.K.V. Iyengar, B. Krishna Gandhi, A Text Book of Engineering Mathematics, Vol – I, S. Chand & Company.

References:

(iv) G.B. Thomas and R.L. Finney, Calculus and Analytic geometry, 9th Edition, Pearson, Reprint, 2002.
(v) Erwin kreyszig, Advanced Engineering Mathematics, 10th Edition, John Wiley & Sons,2011.
(vi) Veerarajan T., Engineering Mathematics for first year, Tata McGraw-Hill, New Delhi, 2008.
(vii) Ramana B.V., Higher Engineering Mathematics, Tata McGraw Hill New Delhi, 11th Reprint, 2010.
(vii) N.P. Bali and Manish Goyal, A text book of Engineering Mathematics, Laxmi Publications, Reprint, 2008.

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Teaching Plan

RGM COLLEGE OF ENGINEERING & TECHNOLOGY, NANDYAL-518 501.

AUTONOMOUS

TEACHING PLAN

NAME OF THE FACULTY: Dr. P. Sudarsana Reddy

Sections: CSE- A

SUBJECT: Linear Algebra and Advanced Calculus

Academic Year: 2020-21

Year: I B.Tech, I-SEM

Total Hours: 61 Hours

S.No	Unit	Topics to be Covered	Estimated Periods
1	I	Elementary row transformations	1
		Definition and Importance of Rank	2
		Echelon form and Normal form	2
		Solutions of Linear System of Homogenous equations.	2
		Solutions of Linear System of Non Homogeneous equations.	3
2	II	Definition and Importance of Eigen Values, Eigen vectors	1
		Calculation of Eigen Values & Eigen vectors	3
		Properties of Eigen Values & Eigen vectors	3
		Cayley – Hamilton Theorem	1
		Inverse and Power of a matrix by Cayley – Hamilton theorem.	2
3	III	Introduction of Quadratic forms	1
		Matrix Representation	1
		Linear Transformation	1
		Reduction of quadratic form to canonical form and their nature (Rank, Signature and Index).	4
4	IV	Introduction of Mean value theorems	1
		Rolle's Theorem	2
		Lagrange's Mean Value Theorem	2
		Simple examples of Taylor's and Maclaurin's Series.	2
		Functions of several variables – Jacobian	2
		Maxima and Minima of functions of two variables	2
		Lagrange method of Multipliers with three variables only.	2
		Introduction of Multiple Integrals	1
		Evaluation of Double Integrals (Including Cartesian and Polar)	2

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5	V	Change of Variables	2
		Change of order of Integration	2
		Changing into Polar coordinates	2
		Evaluation of triple integrals.	2
6	VI	Introduction of Special functions	1
		Gamma function – Properties	3
		Beta function – properties	3
		Relation between Gamma and Beta function	1
		Evaluation of Integrals using Gamma & Beta functions.	3

P. [signature]

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RGM COLLEGE OF ENGINEERING & TECHNOLOGY, NANDYAL-518 501.

AUTONOMOUS

TEACHING PLAN

(Revised)

NAME OF THE FACULTY: Dr. P. Sudarsana Reddy Sections: CSE- A

SUBJECT: **Linear Algebra and Advanced Calculus** Academic Year: 2020-21

Year: I B.Tech, I-SEM Total Hours: 65 Hours

S.No	Unit	Topics to be Covered	Estimated Periods
1	I	Elementary row transformations	1
		Definition and Importance of Rank	2
		Echelon form and Normal form	2
		Solutions of Linear System of Homogenous equations.	2
		Solutions of Linear System of Non Homogeneous equations.	3
2	II	Definition and Importance of Eigen Values, Eigen vectors	1
		Calculation of Eigen Values & Eigen vectors	3
		Properties of Eigen Values & Eigen vectors	3
		Cayley – Hamilton Theorem	1
		Inverse and Power of a matrix by Cayley – Hamilton theorem.	2
3	III	Introduction of Quadratic forms	1
		Matrix Representation	1
		Linear Transformation	2
		Reduction of quadratic form to canonical form and their nature (Rank, Signature and Index).	4
4	IV	Introduction of Mean value theorems	1
		Rolle's Theorem	2
		Lagrange's Mean Value Theorem	2
		Simple examples of Taylor's and Maclaurin's Series.	2
		Functions of several variables – Jacobian	2
		Maxima and Minima of functions of two variables	3
		Lagrange method of Multipliers with three variables only.	3
		Introduction of Multiple integrals	1

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5	V	Evaluation of Double Integrals (Including Cartesian and Polar)	2
		Change of Variables	2
		Change of order of Integration	2
		Changing into Polar coordinates	3
		Evaluation of triple Integrals.	2
6	VI	Introduction of Special functions	1
		Gamma function – Properties	3
		Beta function – properties	3
		Relation between Gamma and Beta function	1
		Evaluation of Integrals using Gamma & Beta functions.	3

***As per the revised plan, another 4more extra hours are required to complete the syllabus of the course.**

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AUTONOMOUS
TEACHING PLAN

NAME OF THE FACULTY: Dr. R. Chandra Sekhar Reddy Sections: CSE-B

SUBJECT: **Linear Algebra and Advanced Calculus** Academic Year: 2020-21

Year: I B.Tech, I-SEM Total Hours: 61 Hours

S.No	Unit	Topics to be Covered	Estimated Periods
1	I	Elementary row transformations	1
		Definition and Importance of Rank	2
		Echelon form and Normal form	2
		Solutions of Linear System of Homogenous equations.	2
		Solutions of Linear System of Non Homogeneous equations.	3
2	II	Definition and Importance of Eigen Values, Eigen vectors	1
		Calculation of Eigen Values & Eigen vectors	3
		Properties of Eigen Values & Eigen vectors	3
		Cayley – Hamilton Theorem	1
		Inverse and Power of a matrix by Cayley – Hamilton theorem.	2
3	III	Introduction of Quadratic forms	1
		Matrix Representation	1
		Linear Transformation	1
		Reduction of quadratic form to canonical form and their nature (Rank, Signature and Index).	4
4	IV	Introduction of Mean value theorems	1
		Rolle's Theorem	2
		Lagrange's Mean Value Theorem	2
		Simple examples of Taylor's and Maclaurin's Series.	2
		Functions of several variables – Jacobian	2
		Maxima and Minima of functions of two variables	2
		Lagrange method of Multipliers with three variables only.	2
		Introduction of Multiple integrals	1
		Evaluation of Double integrals (including Cartesian and Polar)	2

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5	V	Change of Variables	2
		Change of order of Integration	2
		Changing into Polar coordinates	2
		Evaluation of triple integrals.	2
6	VI	Introduction of Special functions	1
		Gamma function – Properties	3
		Beta function – properties	3
		Relation between Gamma and Beta function	1
		Evaluation of Integrals using Gamma & Beta functions.	3

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TEACHING PLAN

(Revised)

(ESTD-1995)

NAME OF THE FACULTY: Dr. R. Chandra Sekhar Reddy   Sections: CSE-B

SUBJECT: **Linear Algebra and Advanced Calculus**   Academic Year: 2020-21

Year: I B.Tech, I-SEM   Total Hours: 65 Hours

S.No	Unit	Topics to be Covered	Estimated Periods
1	I	Elementary row transformations	1
		Definition and Importance of Rank	2
		Echelon form and Normal form	2
		Solutions of Linear System of Homogenous equations.	2
		Solutions of Linear System of Non Homogeneous equations.	3
2	II	Definition and Importance of Eigen Values, Eigen vectors	1
		Calculation of Eigen Values & Eigen vectors	3
		Properties of Eigen Values & Eigen vectors	3
		Cayley – Hamilton Theorem	1
		Inverse and Power of a matrix by Cayley – Hamilton theorem.	2
3	III	Introduction of Quadratic forms	1
		Matrix Representation	1
		Linear Transformation	2
		Reduction of quadratic form to canonical form and their nature (Rank, Signature and Index).	4
4	IV	Introduction of Mean value theorems	1
		Rolle's Theorem	2
		Lagrange's Mean Value Theorem	2
		Simple examples of Taylor's and Maclaurin's Series.	2
		Functions of several variables – Jacobian	2
		Maxima and Minima of functions of two variables	3
		Lagrange method of Multipliers with three variables only.	3
		Introduction of Multiple integrals	1

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5	V	Evaluation of Double integrals (including Cartesian and Polar)	2
		Change of Variables	2
		Change of order of Integration	2
		Changing into Polar coordinates	3
		Evaluation of triple integrals.	2
6	VI	Introduction of Special functions	1
		Gamma function – Properties	3
		Beta function – properties	3
		Relation between Gamma and Beta function	1
		Evaluation of Integrals using Gamma & Beta functions.	3

***As per the revised plan, another 4more extra hours are required to complete the syllabus of the course.**

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TEACHING PLAN

NAME OF THE FACULTY: SG ShakeerHussain　　Sections: CSE- C

SUBJECT: **Linear Algebra and Advanced Calculus**　　Academic Year: 2020-21

Year: I B.Tech, I-SEM　　Total Hours: 61 Hours

S.No	Unit	Topics to be Covered	Estimated Periods
1	I	Elementary row transformations	1
		Definition and Importance of Rank	2
		Echelon form and Normal form	2
		Solutions of Linear System of Homogenous equations.	2
		Solutions of Linear System of Non Homogeneous equations.	3
2	II	Definition and Importance of Eigen Values, Eigen vectors	1
		Calculation of Eigen Values & Eigen vectors	3
		Properties of Eigen Values & Eigen vectors	3
		Cayley – Hamilton Theorem	1
		Inverse and Power of a matrix by Cayley – Hamilton theorem.	2
3	III	Introduction of Quadratic forms	1
		Matrix Representation	1
		Linear Transformation	1
		Reduction of quadratic form to canonical form and their nature (Rank, Signature and Index).	4
4	IV	Introduction of Mean value theorems	1
		Rolle's Theorem	2
		Lagrange's Mean Value Theorem	2
		Simple examples of Taylor's and Maclaurin's Series.	2
		Functions of several variables – Jacobian	2
		Maxima and Minima of functions of two variables	2
		Lagrange method of Multipliers with three variables only.	2
		Introduction of Multiple integrals	1
		Evaluation of Double integrals (including Cartesian and Polar)	2

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5	V	Change of Variables	2
		Change of order of Integration	2
		Changing into Polar coordinates	2
		Evaluation of triple integrals.	2
6	VI	Introduction of Special functions	1
		Gamma function – Properties	3
		Beta function – properties	3
		Relation between Gamma and Beta function	1
		Evaluation of Integrals using Gamma & Beta functions.	3

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AUTONOMOUS

TEACHING PLAN

(Revised)

NAME OF THE FACULTY: SG ShakeerHussain | Sections: CSE- C

SUBJECT: **Linear Algebra and Advanced Calculus** | Academic Year: 2020-21

Year: I B.Tech, I-SEM | Total Hours: 65 Hours

S.No	Unit	Topics to be Covered	Estimated Periods
1	I	Elementary row transformations	1
		Definition and Importance of Rank	2
		Echelon form and Normal form	2
		Solutions of Linear System of Homogenous equations.	2
		Solutions of Linear System of Non Homogeneous equations.	3
2	II	Definition and Importance of Eigen Values, Eigen vectors	1
		Calculation of Eigen Values & Eigen vectors	3
		Properties of Eigen Values & Eigen vectors	3
		Cayley – Hamilton Theorem	1
		Inverse and Power of a matrix by Cayley – Hamilton theorem.	2
3	III	Introduction of Quadratic forms	1
		Matrix Representation	1
		Linear Transformation	2
		Reduction of quadratic form to canonical form and their nature (Rank, Signature and Index).	4
4	IV	Introduction of Mean value theorems	1
		Rolle's Theorem	2
		Lagrange's Mean Value Theorem	2
		Simple examples of Taylor's and Maclaurin's Series.	2
		Functions of several variables – Jacobian	2
		Maxima and Minima of functions of two variables	3
		Lagrange method of Multipliers with three variables only.	3
		Introduction of Multiple integrals	1

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5	V	Evaluation of Double Integrals (Including Cartesian and Polar)	2
		Change of Variables	2
		Change of order of Integration	2
		Changing into Polar coordinates	3
		Evaluation of triple integrals.	2
6	VI	Introduction of Special functions	1
		Gamma function – Properties	3
		Beta function – properties	3
		Relation between Gamma and Beta function	1
		Evaluation of Integrals using Gamma & Beta functions.	3

***As per the revised plan, another 4more extra hours are required to complete the syllabus of the course.**

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AUTONOMOUS

TEACHING PLAN

NAME OF THE FACULTY: K. Jyothi — Sections: EEE-A & B

SUBJECT: **Linear Algebra and Advanced Calculus** — Academic Year: 2020-21

Year: I B.Tech, I-SEM — Total Hours: 61 Hours

S.No	Unit	Topics to be Covered	Estimated Periods
1	I	Elementary row transformations	1
		Definition and Importance of Rank	2
		Echelon form and Normal form	2
		Solutions of Linear System of Homogenous equations.	2
		Solutions of Linear System of Non Homogeneous equations.	3
2	II	Definition and Importance of Eigen Values, Eigen vectors	1
		Calculation of Eigen Values & Eigen vectors	3
		Properties of Eigen Values & Eigen vectors	3
		Cayley – Hamilton Theorem	1
		Inverse and Power of a matrix by Cayley – Hamilton theorem.	2
3	III	Introduction of Quadratic forms	1
		Matrix Representation	1
		Linear Transformation	1
		Reduction of quadratic form to canonical form and their nature (Rank, Signature and Index).	4
4	IV	Introduction of Mean value theorems	1
		Rolle's Theorem	2
		Lagrange's Mean Value Theorem	2
		Simple examples of Taylor's and Maclaurin's Series.	2
		Functions of several variables – Jacobian	2
		Maxima and Minima of functions of two variables	2
		Lagrange method of Multipliers with three variables only.	2
		Introduction of Multiple integrals	1
		Evaluation of Double integrals (including Cartesian and Polar)	2

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5	V	Change of Variables	2
		Change of order of Integration	2
		Changing into Polar coordinates	2
		Evaluation of triple integrals.	2
6	VI	Introduction of Special functions	1
		Gamma function – Properties	3
		Beta function – properties	3
		Relation between Gamma and Beta function	1
		Evaluation of Integrals using Gamma & Beta functions.	3

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AUTONOMOUS

TEACHING PLAN

(Revised)

NAME OF THE FACULTY: K. Jyothi

Sections: EEE-A & B

SUBJECT: **Linear Algebra and Advanced Calculus**

Academic Year: 2020-21

Year: I B.Tech, I-SEM

Total Hours: 65 Hours

S.No	Unit	Topics to be Covered	Estimated Periods
1	I	Elementary row transformations	1
		Definition and Importance of Rank	2
		Echelon form and Normal form	2
		Solutions of Linear System of Homogenous equations.	2
		Solutions of Linear System of Non Homogeneous equations.	3
2	II	Definition and Importance of Eigen Values, Eigen vectors	1
		Calculation of Eigen Values & Eigen vectors	3
		Properties of Eigen Values & Eigen vectors	3
		Cayley – Hamilton Theorem	1
		Inverse and Power of a matrix by Cayley – Hamilton theorem.	2
3	III	Introduction of Quadratic forms	1
		Matrix Representation	1
		Linear Transformation	2
		Reduction of quadratic form to canonical form and their nature (Rank, Signature and Index).	4
4	IV	Introduction of Mean value theorems	1
		Rolle's Theorem	2
		Lagrange's Mean Value Theorem	2
		Simple examples of Taylor's and Maclaurin's Series.	2
		Functions of several variables – Jacobian	2
		Maxima and Minima of functions of two variables	3
		Lagrange method of Multipliers with three variables only.	3
		Introduction of Multiple integrals	1

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5	V	Evaluation of Double integrals (Including Cartesian and Polar)	2
		Change of Variables	2
		Change of order of Integration	2
		Changing into Polar coordinates	3
		Evaluation of triple integrals.	2
6	VI	Introduction of Special functions	1
		Gamma function – Properties	3
		Beta function – properties	3
		Relation between Gamma and Beta function	1
		Evaluation of Integrals using Gamma & Beta functions.	3

***As per the revised plan, another 4more extra hours are required to complete the syllabus of the course.**

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Teaching Diary

LECTURE RECORD

Subject : LA & DC
Credits : 3
Internal Mid Exam Marks : 20
Internal Quiz Marks : 10

Total Exams 2
Each for 2 hrs.
Total quizes
No. of Assignment 2

S.No.	Date	Topic Covered / Exercise Completed	Remarks
1.	15/2/2021	Introduction to Matrices.	UNIT-1
2.	16/2/2021	Applications of matrices in real life.	
3.	18/2/2021	Elementary row operations.	
4.	19/2/21	Def of Rank.	
5.	19/2/2021	Finding rank by Echelon form.	
6.	22/2/2021	Finding rank by Echelon form.	
7.	23/2/2021	Introduction to normal form.	
8.	25/2/2021	Finding rank by normal form.	
9.	26/2/2021	Normal form problems.	
10.	26/2/2021	Solving N-H system of eqn's	
11.	1/3/2021	Solving Non-Homogeneous System of eqn's	
12.	2/3/2021	Solving Homogeneous System of eqn's	UNIT-II
13.	4/3/2021	Homogeneous System	
14.	4/3/2021	Eigen values.	
15.	8/3/2021	Problems related to Eigen values.	
16.	9/3/2021	Eigen values and Vector problems.	
17.	12/3/2021	Eigen values and Vectors problems.	
18.	12/3/2021	Cayley-Hamilton theorem problems.	
19.	15/3/2021	Problems on C-H.	
20.	16/3/2021	Finding A^4, A^3 and A^1 by C-H theorem.	
21.	18/3/2021	Quadratic forms introduction.	UNIT-III
22.	19/3/2021	Converting Q.F to Symmetric form.	
23.	19/3/2021	Converting Q.F to Symmetric form.	
24.	22/3/2021	Symmetric matrix to Q.F.	
25.	25/3/2021	Nature, Signature, index.	
26.	25/3/2021	Reduction of Q.F to Canonical.	
27.	29/3/2021	Reduction of Q.F to Canonical.	
28.	29/3/2021	Reduction of Q.F to Canonical.	
29.	30/3/2021	Reduction of Q.F to Canonical form.	

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LECTURE RECORD

S.No.	Date	Topic Covered / Exercise Completed	Remarks
30.	1/4/2021	Introduction to mean value theorem.	UNIT-IV.
31.	6/4/2021	Rolles theorem def.	
32.	13/4/2021	Problems on Rolles theorem.	
33.	15/4/2021	Lagranges mean value theorem.	
34.	15/4/2021	Lagranges mean value theorem.	
35.	16/4/2021	Taylor's series expansion	
36.	17/4/2021	Taylors series expansion problems.	
37.	19/4/2021	Maclaurins series expansion problems	
38.	22/4/2021	functions of several variables.	
39.	22/4/2021	Jacobian problems.	
40.	23/4/2021	Functional dependant problems.	
41.	26/4/2021	Problems on Functional independant	
ONLINE Classes		– Minimum and maximum of fun of two variables.	
		– Lagranges multipliers with three variables.	
		– Multiple integrals	UNIT-V
		– Double integrals	
		– Region problems on double integrals Cartesian coordinates, polar coordinates	
		– Change of order of integration Cartesian and polar coordinates.	
		– change of variable problems.	
		– Triple integral problems.	
		– Betafuntions	UNIT-V
		– Gamma functions.	
		– Properties.	
		– Relationship b/w Beta and Gamma functions.	

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HOD, [illegible]

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Assignments (I&II) with solutions

Linear Algebra and Advanced Calculus.

Assignment – I

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1. Define the Rank of a Matrix.
2. Explain Echelon form.
3. State Cayley–Hamilton theorem.
4. Write Canonical form of a Quadratic form.
5. Explain Rank, index, signature of the quadratic form.
6. Find rank of $\begin{bmatrix} 1 & 2 & 0 & 3 \\ 1 & -2 & 3 & 0 \\ 0 & 0 & 4 & 8 \\ 2 & 4 & 0 & 6 \end{bmatrix}$ by using normal form.
7. P.T two Eigen vectors corresponding to the two different eigen values are linearly independent.
8. Verify Cayley–Hamilton theorem and find A^{-1}, A^4 of a matrix $A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 2 & 1 & 2 \end{bmatrix}$.
9. Reduce the Q.F into canonical form by diag. and discuss it's nature.
10. Find Eigen values and Eigen vectors for $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

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1. Define the rank of the matrix?

The order of the non-vanishing minor is called the "Rank of matrix".

(or)

If a matrix A is of rank r, then it has to satisfy the two conditions.

i. There is atleast one 'r' rd minor whose determinant $\neq 0$

ii. If there is any $(r+1)^{th}$ rd minor its determinant must be zero.

Example: $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}_{2\times 3}$

The matrix have 2-rd and 1-rd minor.

2. Explain the Echelon form (or) Triangular form of a matrix.

Echelon form (or) Triangular form:

A matrix A is of order $m\times n$, is said to be an Echelon form (or) Triangular form. If its satisfy's the following three conditions.

i. zero row, if any, must below the non-zero rows.

ii. The 1st non-zero element of non-zero row is equal to unity.

iii. The no. of zeros before the non-zero element of a row is less than such zero in the next row. After reducing the matrix to the echlon form the rank of the matrix is

$\rho(A)$ = The number of non-zero rows of a matrix.

Example: (i) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ (iii) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Which is an echelon form.

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3. Explain CAYLEY-HAMILTON THEOREM ?

CAYLEY-HAMILTON THEOREM :

Statement : Every square matrix satisfies it's own charecteristics equation.

Applications of cayley hamilton theorem :

1. To find the inverse of the matrix.
2. To find the powers of a matrix.

4. Write canonical-form of a Quadratic form?

Canonical form of a Quadratic form :

Let $X^T A X$ in Q.F in 'n' variables then their exist a real non-Singular linear transformation $X = P.Y$ which transforms $X^T A X$ to another Quadratic form of the type $Y^T D Y = \lambda_1 y_1^2 + \lambda_2 y_2^2 + ---- --- + \lambda_n y_n^2$ then $Y^T D Y$ is called canonical form of $X^T A X$

Where, D = diagonal of $\lambda_1, \lambda_2, \lambda_3 ------ \lambda_n$.

5. Write the definitions of Rank, Index, & signature of the quadratic form.

Rank of a Quadratic form :

Let, $X^T A X$ be a quadratic form over 'R' the rank of the quadratic form.

* If 'r' is less than n [order of A] or det A = 0 or A is singular then the quadratic form is singular otherwise it is non-Singular.

Index :

The no. of positive terms in canonical (or) normal form of a Quadratic form is known as index of the quadratic form and it is denoted by 'S'.

Signature of the quadratic form :

If 'σ' is the rank of the quadratic form and 's' is the index of the quadratic form then the "excess number of positive terms over the number of negative terms in a normal form" of '$X^T A X$' is called the signature of the quadratic form.

$$\boxed{\text{Signature} = 2s - r}$$

1. Apply normal form to find the rank of the given matrix:

$$\begin{bmatrix} 1 & 2 & 0 & 3 \\ 1 & -2 & 3 & 0 \\ 0 & 0 & 4 & 8 \\ 2 & 4 & 0 & 6 \end{bmatrix}$$

Given, $A \sim \begin{bmatrix} 1 & 2 & 0 & 3 \\ 1 & -2 & 3 & 0 \\ 0 & 0 & 4 & 8 \\ 2 & 4 & 0 & 6 \end{bmatrix}$

$R_2 \to R_2 - R_1$; $R_4 \to R_4 - 2R_1$

$$A \sim \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & -4 & 3 & -3 \\ 0 & 0 & 4 & 8 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_2 \to C_2 - 2C_1$; $C_4 \to C_4 - 3C_1$

$$A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -4 & 3 & -3 \\ 0 & 0 & 4 & 8 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_3 \to \frac{R_3}{4}$

$$A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -4 & 3 & -3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_3 \to 4C_3 + 3C_2$, $C_4 \to 4C_3 - 3C_2$

$$A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -4 & 0 & 0 \\ 0 & 0 & 4 & 8 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_3 \to R_3 / 4$

$$A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -4 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

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$C_4 \to C_4/2$

$$A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -4 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_4 \to C_4 - C_3$

$$A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 \to R_2/-4$

$$A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} \sim \begin{bmatrix} I_3 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\boxed{\rho(A) = 3}$$

2. **Theorem:**

Statement: prove that the two eigen vectors corresponding to the two different eigen values are linearly independent.

proof:

Let 'A' be the Square matrix. Let x_1, x_2 be the two eigen vectors of 'A' corresponding to the two distinct eigen values λ_1 and λ_2 then

$$AX_1 = \lambda_1 X_1 \longrightarrow (1)$$

$$AX_2 = \lambda_2 X_2 \longrightarrow (2)$$

Now, we shall prove that the eigen vectors X_1 and X_2 are linearly independent.

Let us assume that x_1 and x_2 are linearly dependent then for two Scalars k_1 and k_2 not both zero, Such that $k_1 X_1 + k_2 X_2 = 0 \rightarrow (3)$

Multiply eq (3) by 'A'

$$A[k_1 X_1 + k_2 X_2] = 0$$

$$k_1(AX_1) + k_2(AX_2) = 0$$

$$k_1(\lambda_1 X_1) + k_2(\lambda_2 X_2) = 0 \longrightarrow (4)$$

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Multiply eq ③ by 'λ_1'

$k_1\lambda_1x_1 + k_2\lambda_2 x_2 = 0 \longrightarrow ⑤$

Subtract eq ⑤ - eq ④

$$\cancel{k_1\lambda_1 x_1} + k_2\lambda_1 x_2 = 0$$
$$\cancel{k_1\lambda_1 x_1} + k_2\lambda_2 x_2 = 0$$

$$k_2\lambda_1 x_2 - k_2\lambda_2 x_2 = 0$$
$$k_2(\lambda_1-\lambda_2)x_2 = 0$$
$$x_2 \neq 0, \ \lambda_1 - \lambda_2 \neq 0$$

But this is a contradiction. for our assumption that k_1 and k_2 are not zeroes.

Hence our assumption that x_1 and x_2 are linearly dependent is wrong.

Hence the statement is true.

∴ Hence, the theorem is proved.

3. Verify CH theorem. Find A^{-1} and A^4 of the matrix $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 2 & 1 & 2 \end{bmatrix}$

$(A-\lambda I)X = 0$

$$\begin{bmatrix} 1-\lambda & -1 & 0 \\ 0 & 1-\lambda & 1 \\ 2 & 1 & 2-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

The charecteristic equation of A is $|A-\lambda I| = 0$

i.e.,

$$\begin{vmatrix} 1-\lambda & -1 & 0 \\ 0 & 1-\lambda & 1 \\ 2 & 1 & 2-\lambda \end{vmatrix} = 0$$

$$1-\lambda[(1-\lambda)(2-\lambda)-1] + 1[0-2] = 0$$
$$1-\lambda[2-\lambda-2\lambda+\lambda^2-1] + 0 - 2 = 0$$
$$1-\lambda[\lambda^2-3\lambda+1] + 0 - 2 = 0$$
$$-2\lambda^2-3\lambda+1-\lambda^3+3\lambda^2-\lambda = 0$$
$$-2-\lambda^3+4\lambda^2-4\lambda+1 = 0$$
$$+2+\lambda^3-4\lambda^2+4\lambda-1 = 0$$
$$\lambda^3-4\lambda^2+4\lambda+1 = 0 \longrightarrow ①$$

To verify CH theorem put $\lambda = A$ and multiply constant by I, then we get.

$$A^3-4A^2+4A+I=0 \rightarrow (2)$$

$$\begin{bmatrix}1&-1&0\\0&1&1\\2&1&2\end{bmatrix}\begin{bmatrix}1&-1&0\\0&1&1\\2&1&2\end{bmatrix}\begin{bmatrix}1&-1&0\\0&1&1\\2&1&2\end{bmatrix}-4\begin{bmatrix}1&-1&0\\0&1&1\\2&1&2\end{bmatrix}\begin{bmatrix}1&-1&0\\0&1&1\\2&1&2\end{bmatrix}+4\begin{bmatrix}1&-1&0\\0&1&1\\2&1&2\end{bmatrix}+\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=0$$

$$\begin{bmatrix}1-0+0&-1-1+0&0-1+0\\0+0+2&0+1+1&0+1+2\\2+0+4&-2+1+2&0+1+4\end{bmatrix}\begin{bmatrix}1&-1&0\\0&1&1\\2&1&2\end{bmatrix}-4\begin{bmatrix}1&-1&0\\0&1&1\\2&1&2\end{bmatrix}\begin{bmatrix}1&-1&0\\0&1&1\\2&1&2\end{bmatrix}+\begin{bmatrix}4&-4&0\\0&4&4\\8&4&8\end{bmatrix}+\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=0$$

$$\begin{bmatrix}-1&-4&-4\\8&3&8\\16&0&11\end{bmatrix}-4\begin{bmatrix}1&-2&-1\\2&2&3\\6&1&4\end{bmatrix}+\begin{bmatrix}4&-4&0\\0&4&4\\8&4&8\end{bmatrix}+\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix};$$

$\therefore$ CH theorem is verified. So, we can calculate A^4 & A^{-1}

From (2), $A^3-4A^2+4A+I=0$

Multiply with 'A' on both sides

$$A^4-4A^3+4A^2+I.A=0$$

$$A^4=4A^3-4A^2-IA=0 \qquad I.A=A$$

$$A^4=4A^3-4A^2-A=0$$

$$A^4=4\begin{bmatrix}-1&-4&-4\\8&3&8\\16&0&11\end{bmatrix}-4\begin{bmatrix}1&-2&1\\2&2&3\\6&1&5\end{bmatrix}-1\begin{bmatrix}1&-1&0\\0&1&1\\2&1&2\end{bmatrix}$$

$$A^4=\begin{bmatrix}-4&-16&-16\\32&12&32\\64&0&44\end{bmatrix}-\begin{bmatrix}4&-8&-4\\8&8&12\\24&4&20\end{bmatrix}-\begin{bmatrix}1&-1&0\\0&1&1\\2&1&2\end{bmatrix}$$

$$A^4=\begin{bmatrix}-4-4-1&-16+8+1&-16+4-0\\32-8-0&12-8-1&32-12-1\\64-24-2&0-4-1&44-20-2\end{bmatrix}$$

$$A^4=\begin{bmatrix}-9&-7&-12\\24&13&19\\38&-5&22\end{bmatrix}$$

$$A^3-4A^2+4A+I=0$$

$$I=A^3+4A^2-4A$$

$$A^{-1}=-\begin{bmatrix}1&-2&-1\\2&2&3\\6&1&5\end{bmatrix}+4\begin{bmatrix}1&-1&0\\0&1&1\\2&1&2\end{bmatrix}-4\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

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$$= \begin{bmatrix} -1 & 2 & 1 \\ -2 & -2 & -3 \\ -6 & -1 & -5 \end{bmatrix} + \begin{bmatrix} 4 & -4 & 0 \\ 0 & 4 & 4 \\ 8 & 4 & 8 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -1+4-4 & 2-4-0 & 1+0-0 \\ -2+0-0 & -2+4-4 & -3+4-0 \\ -6+8-0 & -1+4+0 & -5+8-4 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -2 & 1 \\ -2 & -2 & 1 \\ 2 & 3 & -1 \end{bmatrix}$$

4\. Reduce the Q.F into canonical form by diagonalization and discuss the nature of Q.F?

$$2x^2+2y^2+2z^2-2xy-2yz-2zx$$

$$\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

$$A_{3\times3} = I_{3\times3} A\, I_{3\times3}$$

$$\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 \to 2R_2 + R_1$; $R_3 \to 2R_3 + R_1$

$$\begin{bmatrix} 2 & -1 & -1 \\ 0 & 3 & -3 \\ 0 & -3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$C_2 \to 2C_2 + C_1$; $C_3 \to 2C_3 + C_1$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & -3 \\ 0 & -3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} A \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & -3 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 2 & 2 & 2 \end{bmatrix} A \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$C_3 \to C_3 + C_2$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 2 & 2 & 2 \end{bmatrix} A \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ 0 & 0 & 2 \end{bmatrix}$$

$$D = P^T A P$$

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$$D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix}; \quad P = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ 0 & 0 & 2 \end{bmatrix}$$

Rank $= 2$

index $s = 2$

Signature $= 2s - r$
$= 2(2) - 2$
$= 4 - 2$
$= 2$

Nature = +ve semi definate

conical form $= Y^T D Y$

$$Y^T D Y = [y_1, y_2 \, y_3] \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$Y^T D Y = 2y_1^2 + 3y_2^2 + 0y_3^2$$

$X = PY$ [linear transformation]

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$x_1 = y_1 + y_2 + 2y_3$$

5) Find the eigen values and eigen vectors for $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

Given matrix

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$(A - \lambda I)X = 0$$

$$\left(\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} 1-\lambda & 1 & 1 \\ 1 & 1-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 1 & 1 \\ 1 & 1-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{vmatrix} = 0$$

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$1-\lambda[(1-\lambda)(1-\lambda)-1]-1[(1-\lambda)-1]+1[1-(1-\lambda)]$

$1-\lambda[1-\lambda-\lambda+\lambda^2-1]-1[1-\lambda+1]+1[1-1+\lambda]$

$1-\lambda[1-2\lambda+\lambda^2-1]-1[1-\lambda-1]+1[1-1+\lambda]$

$3\lambda^2-\lambda^3=0$

$\lambda^2(3-\lambda)=0$

$\lambda^2=0 \quad 3-\lambda=0$

$\lambda=0,0 \quad \lambda=3$

$\therefore$ Eigen values $\lambda=0,0,3$

Eigen vectors corresponding to the eigen value $\lambda=0$

$$\begin{bmatrix}1-0 & 1 & 1\\ 1 & 1-0 & 1\\ 1 & 1 & 1-0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=0$$

$$\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=0$$

$R_2\to R_2-R_1 ; R_3\to R_3-R_1$

$$\begin{bmatrix}1 & 1 & 1\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=0$$

$r=1<n$

$n-r=3-1=2$

$x+y+z=0$

$y=k_1 ; z=k_2$

$x+y+z=0$

$k_1+k_2+x=0$

$x=-(k_1+k_2)$

$$X_1=\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}-(k_1+k_2)\\k_1\\k_2\end{bmatrix}$$ is a vector of eigen value of '0'

Eigen vector corresponding to the eigen value $\lambda=3$

$$\begin{bmatrix}1-3 & 1 & 1\\ 1 & 1-3 & 1\\ 1 & 1 & 1-3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=0$$

$$\begin{bmatrix}-2 & 1 & 1\\ 1 & -2 & 1\\ 1 & 1 & -2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=0$$

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$R_2 \to 2R_2 + R_1$; $R_3 \to 2R_3 + R_1$

$$\begin{bmatrix} -2 & 1 & 1 \\ 0 & -3 & 3 \\ 0 & 3 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$R_3 \to R_3 + R_2$

$$\begin{bmatrix} -2 & 1 & 1 \\ 0 & -3 & 3 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$r = 2 < n$

$n - r = 3 - 2 = 1$

$-2x + y + z = 0$

$-3y + 3z = 0$

Let $\boxed{z = k}$

$-3y + 3(k) = 0$

$-3y = -3k$

$\boxed{y = k}$

$-2x + k + k = 0$

$\boxed{x = k}$.

$x = k$; $y = k$; $z = k$

$$x_2 = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} k \\ k \\ k \end{bmatrix} = k\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$x_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is a vector of eigen value of 3.

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Linear Algebra and Advanced calculus. R-20 32

Assignment - II

1.(a) Show that $\int_0^1 \frac{1}{\sqrt{1-x^n}} \cdot dx = \frac{\sqrt{\pi}}{n} \cdot \frac{\Gamma(1/n)}{\Gamma(1/n+1/2)}$

(b). S.T $\int_0^1 \frac{1}{(1-x^n)^{1/n}} \cdot dx = \frac{\pi}{n} \operatorname{cosec}(\pi/n)$

2(a) Verify Rolles theorem for the function $f(x)=(x-a)^m(x-b)^n$ where m, n are +ve integers.

(b) If $x = r\cos\phi\sin\theta$, $y = r\sin\theta\sin\phi$. $z = r\cos\theta$ then S.T $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)} = r^2\sin\theta$.

3. Evaluate $\iiint xy^2z \, dx\,dy\,dz$ taken through the +ve octant of the sphere $x^2+y^2+z^2=a^2$.

4(a) S.T $\Gamma(1/2) = \sqrt{\pi}$

(b) $\int_a^b (x-a)^m (b-x)^n \cdot dx = (b-a)^{m+n+1} \beta(m+1, n+1)$.

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Assignment-2

1) a) Show that : $\int_0^1 \frac{1}{\sqrt{(1-x^n)}}\,dx = \frac{\sqrt{\pi}}{n}\,\frac{\Gamma\left(\frac{1}{n}\right)}{\Gamma\left(\frac{1}{n}+\frac{1}{2}\right)}$

$$\int_0^1 \frac{1}{\sqrt{(1-x^n)}}\,dx = \frac{\sqrt{\pi}}{n}\,\frac{\Gamma\left(\frac{1}{n}\right)}{\Gamma\left(\frac{1}{n}+\frac{1}{2}\right)}$$

$$x^n = y$$

$$x = y^{1/n}$$

$$dx = \frac{1}{n}\,y^{1/n-1}\,dy$$

$$= \int_0^1 \frac{1}{1-y^{-1/2}}\cdot\frac{1}{n}\cdot y^{1/n-1}\,dy$$

$$= \int_0^1 (1-y)^{-1/2}\cdot\frac{1}{n}\cdot y^{\frac{1}{n}-1}\,dy$$

$$= \frac{1}{n}\int_0^1 (1-y)^{\left(1-\frac{1}{2}\right)-1}\cdot y^{1/n-1}\,dy$$

$$= \frac{1}{n}\int_0^1 y^{1/n-1}\cdot(1-y)^{\left(1-\frac{1}{2}\right)-1}\,dy$$

$$B(m,n) = \int_0^1 x^{m-1}\,(1-x)^{n-1}\,dx$$

Now,

$$= \frac{1}{n}\,B\left(\frac{1}{n},\ 1-\frac{1}{2}\right)$$

$$= \frac{1}{n}\,B\left(\frac{1}{n},\ \frac{2+1}{2}\right)$$

$$= \frac{1}{n}\,B\left(\frac{1}{n},\ \frac{1}{2}\right)$$

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$$B(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$

$$= \frac{1}{n} \frac{\Gamma\left(\frac{1}{n}\right)\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{1}{n}+\frac{1}{2}\right)} \quad \therefore \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

$$= \frac{\sqrt{\pi}}{n} \cdot \frac{\Gamma(1/n)}{\Gamma\left(\frac{1}{n}+\frac{1}{2}\right)}$$

$\therefore$ L.H.S = R.H.S.

b) show that $\int_0^1 \frac{1}{(1-x^n)^{1/n}} dx = \frac{\pi}{n} \operatorname{cosec}\left(\frac{\pi}{n}\right)$

$x^n = y$

$x = y^{1/n}$

$dx = \frac{1}{n} y^{1/n-1} dy$

$$= \int_0^1 \frac{1}{(1-y)^{1/n}} \cdot \frac{1}{n} \cdot y^{\frac{1}{n}-1} dy$$

$$= \frac{1}{n} \int_0^1 (1-y)^{-1/n} y^{\frac{1}{n}-1} \cdot dy$$

$$= \frac{1}{n} \int_0^1 (1-y)^{\left(1-\frac{1}{n}\right)-1} \cdot y^{\frac{1}{n}-1} dy$$

$$= \frac{1}{n} \int_0^1 y^{\frac{1}{n}-1} \cdot (1-y)^{\left(1-\frac{1}{n}\right)-1}$$

$$B(m,n) = \int_0^1 x^{m-1} (1-x)^{n-1} dx$$

$$= 1/n \; B\left(\frac{1}{n}, 1-\frac{1}{n}\right)$$

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$$= \frac{1}{n} \cdot \frac{\Gamma\left(\frac{1}{n}\right)\Gamma\left(1-\frac{1}{n}\right)}{\Gamma\left(\frac{1}{n}+1-\frac{1}{n}\right)} \qquad \because \Gamma(1) = 1.$$

$$= \frac{1}{n} \cdot \Gamma\left[\frac{1}{n}\right] \cdot \Gamma\left[1-\frac{1}{n}\right]$$

$$= \frac{1}{n} \frac{\pi}{\sin\frac{1}{n}\pi} = \frac{1}{n} \cdot \frac{\pi}{\sin\frac{\pi}{n}}$$

$$= \frac{\pi}{n} \cdot \operatorname{cosec}\left(\frac{\pi}{n}\right)$$

L.H.S. = R.H.S.

2) a) verify Rolle's theorem for the function $f(x) = (x-a)^m (x-b)^n$, where m, n are positive integers in $[a, b]$.

b) If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$ then show that

$$\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2\sin\theta.$$

a) Given

$f(x) = (x-a)^m (x-b)^n$ in $[a, b]$

i) since it is a polynomial in x

$\therefore$ it is continous in the closed $[a, b]$

ii) $f'(x) = \frac{d}{dx}\left[(x-a)^m (x-b)^n\right]$

$$= (x-a)^m n(x-b)^{n-1}(1) + (x+b)^n m(x-a)^{m-1}$$

$$= (x-a)^{m-1}(x-b)^{n-1}\left[x(m+n) - (na+mb)\right] \text{ is exist}$$

$\therefore$ It is derivable in (a, b)

iii) $f(a) = 0$, $f(b) = 0$

$f(a) = f(b)$

$\therefore f(a) = f(b)$

then at least one point $c \in (a, b)$

$f'(c) = 0$

Let $f'(c) = 0$

$[(c-a)^{m-1}(c-b)^{n-1}[c(m+n) - (na+mb)] = 0$

$c(m+n) - (na+mb) = 0$

$$c = \frac{na+mb}{m+n}$$

Hence the point c divides a, b internally in the ratio of m : n

Hence the rolles theorem is proofed.

b) $$\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{dx}{\partial \theta} & \frac{\partial x}{\partial \phi} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} & \frac{\partial y}{\partial \phi} \\ \frac{\partial z}{\partial r} & \frac{\partial z}{\partial \theta} & \frac{\partial z}{\partial \phi} \end{vmatrix}$$

$\cancel{\frac{\partial x}{\partial r} = \sin\theta \cos\phi}$

$\cancel{\frac{\partial x}{\partial \theta} = 0}$

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$$\begin{vmatrix} \sin\theta\cos\phi & r\cos\theta\cos\phi & -r\sin\theta\sin\phi \\ \sin\theta\sin\phi & r\cos\theta\sin\phi & r\sin\theta\cos\phi \\ \cos\theta & -r\sin\theta & 0 \end{vmatrix}$$

$= \sin\theta\cos\phi[r\sin\theta\cos\theta) + r\sin\theta)] - r\cos\theta\cos\phi$
$[-(r\sin\theta\cos\phi)(\cos\theta)] - r\sin\theta\sin\phi[(\sin\theta$
$\sin\phi) - (-r\sin\theta) - r(\cos\theta\sin\phi)(\cos\theta)]$

$= \sin\theta\cos\phi(r^2\sin^2\theta\cos\phi) - r\cos\theta\cos\phi)$
$(-r\sin\theta\cos^2\theta\cos\phi) - r\sin\theta\sin\phi$
$[(-r\sin\theta\sin\phi) - r\cos^2\theta\sin\phi)]$

$= r^2\sin^3\theta(\cos^2\phi + \sin^2\phi] + r^2\cos^2\theta[\sin\theta$
$\cos^2\phi + \sin^2\phi\sin\theta]$

$= r^2\sin^3\theta(1) + r^2\cos^2\theta\sin\theta$

$= r^2\sin\theta[\sin^2\theta + \cos^2\theta]$

$= r^2\sin\theta$

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3) Evaluate the triple integral $\iiint xy^2z \, dx\,dy\,dz$ taken through the positive octant of the sphere $x^2+y^2+z^2=a^2$

$\iiint xy^2z \, dx\,dy\,dz$

$x^2+y^2+z^2=a^2$

$z = 0 \text{ to } \sqrt{a^2-x^2-y^2}$

$y = 0 \text{ to } \sqrt{a^2-x^2}$

$x = 0 \text{ to } a$

$\iiint xy^2z \, dx\,dy$

$$= \int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y}} xy^2z \, dz\,dy\,dx$$

$$= \int_0^a \int_0^{\sqrt{a^2-x^2}} xy^2 \left[\frac{z^2}{2}\right]_0^{\sqrt{a^2-x^2-y}} dy\,dx$$

$$= \frac{1}{2}\int_0^a \int_0^{\sqrt{a^2-x^2}} xy^2 (a^2-x^2-y) \, dy\,dx.$$

$$= \frac{1}{2}\int_0^a \int_0^{\sqrt{a^2-x^2}} (a^2xy^2 - x^2y) \, dy\,dx.$$

$$= \frac{1}{2}\int_0^a \int_0^{\sqrt{a^2-x^2}} x(a^2y^2 - x^2y^2 - y^4) \, dy\,dx$$

$$= \frac{1}{2}\int_0^a \int_0^{\sqrt{a^2-x^2}} x(a^2-x^2)y^2 - y^4) \, dy\,dx.$$

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$$= \frac{1}{2}\int_0^a x\left[(a^2-x^2)\left(\frac{y^3}{3}\right) - \left(\frac{y^5}{5}\right)\right] dx.$$

$$= \frac{1}{2}\int_0^a x\left[(a^2-x^2)\left[\frac{y^3}{3}\right]_0^{\sqrt{a^2-x^2}} - \left[\frac{y^5}{5}\right]_0^{\sqrt{a^2-x^2}}\right] dx.$$

$$= \frac{1}{2}\int_0^a x (a^2-x^2)\, \frac{1}{3}(a^2-x^2)^{3/2} - \frac{1}{5}(a^2-x^2)^{5/2}\Big]$$

$$= \frac{1}{2}\int_0^a x\, (a^2-x^2)^{5/2}\, \frac{1}{3} - \frac{1}{5}(a^2-x^2)^{5/2}\Big]\, dx.$$

$$= \frac{1}{2}\int_0^a (a^2-x^2)^{5/2}\left[\frac{1}{3}-\frac{1}{5}\right] dx.$$

$$= \frac{1}{2}\int_0^a (a^2-x^2)^{5/2} \cdot x\, \frac{2}{15}\, dx.$$

$$= \frac{1}{15}\int_0^a x(a^2-x^2)^{5/2}\, dx$$

put $a^2-x^2 = t$

$-2x\,dx = dt$

$x\,dx = \frac{-dt}{2}$

$$= \frac{1}{15}\int_0^a (t)^{5/2}\, \frac{-dt}{2}$$

$$= \frac{-1}{30}\int_0^a (t)^{5/2}\, dt$$

$$= \frac{-1}{30}\left(\frac{t^{7/2}}{7/2}\right)_0^a$$

$$= \frac{-1}{30\times 7}\left[(a^2-x^2)^{7/2}\right]_0^a$$

$$= \frac{-a^7}{105}$$

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4) a) show that $\Gamma(\frac{1}{2}) = \sqrt{\pi}$.

b) $\int_a^b (x-a)^m (b-x)^n dx = (b-a)^{m+n+1}$

$\beta(m+1, n+1)$.

Let equation of β, γ function is

$$B(m, n) = \frac{\gamma(m)\gamma(n)}{\gamma(m+n)} \rightarrow ①$$

here $m=1$, $n=1$

$$B(\tfrac{1}{2}, \tfrac{1}{2}) = \frac{\gamma(1/2)\gamma(1/2)}{\gamma(1/2+1/2)}$$

$$B(\tfrac{1}{2}, \tfrac{1}{2}) = [\gamma(1/2)]^2 \rightarrow ②$$

Let the eq of B is

$$B = (x^{m-1})(1-x)^{n-1} dx \rightarrow ③$$

$$B = (x^{1/2-1})(1-x)^{1/2-1} dx$$

$$B = \frac{1}{x^{1/2}} \cdot \frac{1}{(1-x)^{1/2}} dx$$

$$B(1/2, 1/2) = \frac{1}{x^{1/2}} \cdot \frac{1}{(1-x)^{1/2}} dx.$$

Let

$x = \sin^2\theta$ $\quad 1-x = \cos^2\theta$ $\quad dx = 2\sin\theta\cos\theta\, d\theta$.

If $x=0$ the $\sin^2\theta = 0$

$$\boxed{\theta = 0}$$

If $x=1$ then $\sin^2\theta = 1$

$\sin\theta = 1$

$$\boxed{\theta = \pi/2}$$

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$$B(1/2, 1/2) = \int_0^{\pi/2} \frac{1}{\sin\theta} \frac{1}{\cos\theta} 2\sin\theta\cos\theta \, d\theta$$

$$= 2\int_0^{\pi/2} 1 \, d\theta$$

$$= 2[\theta]_0^{\pi/2}$$

$$= 2\left[\frac{\pi}{2} - 0\right]$$

$$= 2 \times \frac{\pi}{2}$$

$$B(1/2, 1/2) = \pi \rightarrow (4)$$

from eq (2) and (4)

L.H.S = R.H.S

b) Let

$x = a + (b-a)y$

$dx = (b-a)\,dy$

If $x = a \Rightarrow a = a + (b-a)y$

$\boxed{y = 0}$

If $x = b \Rightarrow b = a + (b-a)y$

$\boxed{y = 1}$

$$\int_a^b (x-a)^m (b-x)^n \, dx.$$

$$= \int_0^1 (a + (b-a)y - a)^m (b - a + (b-a)y)^n \ldots$$

$$= \int_0^1 (b-a)y^m ((b-a) - (b-a)y]^n (b-a)\,dy$$

$$= \int_0^1 (b-a)^{m+1} y^m (b-a) (1-y)^n dy$$

$$= \int_0^1 (b-a)^{m+n+1} \int_0^1 y^n (1-y)^n dy$$

$$= (b-a)^{m+n+1} \int_0^1 y^n (1-y)^n dy$$

$$= (b-a)^{m+n+1} \int_0^1 x^m (1-x)^n dx$$

$$= (b-a)^{m+n+1} B(m+1)(n+1)$$

Hence proved.

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Mid Examination (I&II) Papers with solutions

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College Code: 09

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I B. Tech. I-Semester Mid-I *Examinations*

Linear Algebra & Advanced Calculus

(CE, EEE, ME, CSE, CSE (DS), CSE & BS)

Max. Marks: 20. Date: 07 -04 - 2021. Time: 2 Hours

Note: 1. Answer first question compulsorily. (5 x 1 = 05 Marks)

2. Answer any *three* from 2 to 5 questions (5 x 3 = 15 Marks)

1) i) Define rank of a matrix and explain it with an example.

ii) Find rank of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$.

iii) Write the statement of Cayley – Hamilton theorem.

iv) Find Eigen values of the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

v) Write the quadratic form corresponding to the matrix $\begin{bmatrix} 1 & 2 & -3 \\ 2 & -1 & 4 \\ -3 & 4 & 2 \end{bmatrix}$.

2) a) Find rank of a matrix $\begin{bmatrix} 2 & 1 & 5 & 1 \\ -1 & 2 & 5 & 3 \\ 3 & 2 & 9 & -1 \end{bmatrix}$ by reducing into Echelon form.

b) Solve the following system of equations, $x + y - 3z + 2w = 0$;

$x + 2y + z = 0$; $3x + 4y + 4z = 0$; $7x + 10y + 12z = 0$.

3) a) Solve the following system of equations,

$3x + 2y + z = 3$; $2x + y + z = 0$; $6x + 2y + 4z = 6$.

b) Verify Cayley – Hamilton theorem for the matrix $A = \begin{bmatrix} 8 & -8 & 2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$.

4) Find Eigen values and Eigen vectors of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.

5) a) Find the Rank, Index, Signature and Nature of the quadratic from

$x^2 + 2y^2 + 3z^2 + 2yz - 2zx + 2xy$.

b) If $\lambda_1, \lambda_2 \ldots \lambda_n$, are the Eigen values of a matrix A then prove that

$1/\lambda_1, 1/\lambda_2 \ldots 1/\lambda_n$, are the Eigen values of A^{-1}.

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I B.Tech I-Semester Mid-I Examinations.

Linear Algebra and Advanced Calculus.

Max Marks : 20 Date : 09-04-2021. Time : 2 Hours.

1. Answer first question compulsorily. (5×1 = 05 marks)
2. Answer any three from 2 to 5 questions. (5×3 = 15 marks)

1. (i) Define rank of a matrix and explain with an example.

Rank of a matrix:

The highest order of a non vanishing ($\neq 0$) minor of a matrix is called a rank of the matrix. (or)

If a matrix A is of rank r, then it has to satisfy the two conditions.

(i) There is atleast one 'r' rowed minor whose determinant $\neq 0$.

(ii) If there is any $(r+1)^{th}$ rowed minor it's determinant must be zero

Ex: 1) $\begin{bmatrix} 1 & 2 & 3 \\ 6 & 5 & 4 \\ 2 & 0 & 0 \end{bmatrix}$ 2) $\begin{bmatrix} 4 & 5 & 1 \\ 6 & 2 & 2 \\ 0 & 0 & 0 \end{bmatrix}$ 3) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ 4) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\rho(A) = 3$, $\rho(A) = 2$, $\rho(A) = 3$, $\rho(A) = 1$.

2) Find the rank of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$

Given $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore \rho(A) = 1$.

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(iii) Write the statement of Cayley-Hamilton theorem.

Every square matrix satisfies its own characteristic equations.

(iv) Find the eigen values of the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

Given $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

The given matrix is a triangular matrix. We know that for a triangular matrix the Eigen values are the diagonal elements.

$\therefore$ The Eigen values are 1, 2, 3.

(v) Write the quadratic form corresponding to the matrix $\begin{bmatrix} 1 & 2 & -3 \\ 2 & -1 & 4 \\ -3 & 4 & 2 \end{bmatrix}$

Given matrix is $\begin{bmatrix} 1 & 2 & -3 \\ 2 & -1 & 4 \\ -3 & 4 & 2 \end{bmatrix}$

Quadratic form of a matrix is

$$x^2 + 4xy - 6xz - y^2 + 8yz + 2z^2 = Q.$$

2(a) Find rank of a matrix $\begin{bmatrix} 2 & 1 & 5 & 1 \\ -1 & 2 & 5 & 3 \\ 3 & 2 & 9 & -1 \end{bmatrix}$ by reducing into Echelon form.

Given matrix $\begin{bmatrix} 2 & 1 & 5 & 1 \\ -1 & 2 & 5 & 3 \\ 3 & 2 & 9 & -1 \end{bmatrix}$

$R_2 \to 2R_2 + R_1$

$R_3 \to 2R_3 - 3R_1$

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$$\Rightarrow \begin{bmatrix} 2 & 1 & 5 & 1 \\ 0 & 5 & 15 & 7 \\ 0 & 1 & 3 & -5 \end{bmatrix}$$

$R_3 \to 5R_3 - R_2$

$$\Rightarrow \begin{bmatrix} 2 & 1 & 5 & 1 \\ 0 & 5 & 15 & 7 \\ 0 & 0 & 0 & -32 \end{bmatrix}$$

It is reduced in the echelon form and rank of the matrix $\rho(A) = 3$.

(b) Solve the following system of eqn's $x+y-3z+2w=0$, $x+2y+z=0$, $3x+4y+4z=0$, $7x+10y+12z=0$.

Given system of eqn's are homogeneous eqn's

$$x+y-3z+2w=0.$$
$$x+2y+z+0=0$$
$$3x+4y+4z+0=0$$
$$7x+10y+12z+0=0.$$

eqn's can be written in matrix form as

$$\begin{bmatrix} 1 & 1 & -3 & 2 \\ 1 & 2 & 1 & 0 \\ 3 & 4 & 4 & 0 \\ 7 & 10 & 12 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - 3R_1$, $R_4 \to R_4 - 7R_1$

$$\begin{bmatrix} 1 & 1 & -3 & 2 \\ 0 & 1 & 4 & -2 \\ 0 & 1 & 13 & -6 \\ 0 & 3 & 33 & -14 \end{bmatrix}$$

$R_3 \to R_3 - R_2$, $R_4 \to R_4 - 3R_2$

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$$\begin{bmatrix} 1 & 1 & -3 & 2 \\ 0 & 1 & 4 & -2 \\ 0 & 0 & 9 & -4 \\ 0 & 0 & 21 & -8 \end{bmatrix}$$

$R_4 \to 9R_4 - 21R_3$

$$\begin{bmatrix} 1 & 1 & -3 & 2 \\ 0 & 1 & 4 & -2 \\ 0 & 0 & 9 & -4 \\ 0 & 0 & 0 & 12 \end{bmatrix}$$

Rank of $A \Rightarrow \rho(A) = 4$.

no. of unknowns $n = 4$.

$\therefore \rho(A) = n = 4$.

$\therefore$ $x + y - 3z + 2w = 0$

$y + 4z - 2w = 0$

$9z - 4w = 0$

$12w = 0$

$\therefore w = 0,\ z = 0,\ y = 0,\ x = 0.$

3(a) Solve the following system of equations.

$3x + 2y + z = 3,\quad 2x + y + z = 0,\quad 6x + 2y + 4z = 6.$

Given eqns can be written in the matrix form as

$$\begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 1 \\ 6 & 2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 6 \end{bmatrix} \Leftrightarrow AX = B$$

Augmented matrix $[AB]$

$$\begin{bmatrix} 3 & 2 & 1 & 3 \\ 2 & 1 & 1 & 0 \\ 6 & 2 & 4 & 6 \end{bmatrix}$$

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$R_2 \to 3R_2 - 2R_1$

$R_3 \to R_3 - 2R_1$

$$\Rightarrow \begin{bmatrix} 3 & 2 & 1 & 3 \\ 0 & -1 & 1 & -6 \\ 0 & -2 & 2 & 0 \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$

$$\begin{bmatrix} 3 & 2 & 1 & 3 \\ 0 & -1 & 1 & -6 \\ 0 & 0 & 0 & 12 \end{bmatrix}$$

$\rho(A) = 3, \quad \rho(AB) = 4.$

$\rho(A) \neq \rho(AB)$

The given system of eqn's are in consistent.

3(b) Verify Cayley-Hamilton theorem for the matrix

$$A = \begin{bmatrix} 8 & -8 & 2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$$

Given $A = \begin{bmatrix} 8 & -8 & 2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$

Char: eqn is $|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 8-\lambda & -8 & 2 \\ 4 & -3-\lambda & -2 \\ 3 & -4 & 1-\lambda \end{vmatrix} = 0.$$

$\Rightarrow -\lambda^3 + 6\lambda^2 + \lambda - 22 = 0. \longrightarrow (1)$

Put $\lambda = A$ in eq (1) then we get.

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$$\Rightarrow -A^3+6A^2+A-22I=0.$$

$$\Rightarrow A^3=\begin{bmatrix}214 & -296 & 206\\ 88 & -115 & 70\\ 69 & -100 & 69\end{bmatrix}, \quad A^2=\begin{bmatrix}38 & -48 & 34\\ 14 & -15 & 12\\ 11 & -16 & 15\end{bmatrix}.$$

$$\Rightarrow \begin{bmatrix}-214 & 296 & -206\\ -88 & 115 & -70\\ -69 & 100 & -69\end{bmatrix}+\begin{bmatrix}288 & -288 & 204\\ 84 & -90 & 72\\ 66 & -96 & 90\end{bmatrix}+\begin{bmatrix}-14 & -8 & 2\\ 4 & -25 & -2\\ 3 & -4 & -21\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$\therefore$ Cayley Hamiton theorm is veified.

(4) Find Eigen values and Eigen vectors of the matrix

$$\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}$$

Given matrix $A=\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}$

$|A-\lambda I|=0$

$$\Rightarrow \begin{vmatrix}6-\lambda & -2 & 2\\ -2 & 3-\lambda & -1\\ 2 & -1 & 3-\lambda\end{vmatrix}=0$$

$$\Rightarrow -\lambda^3+12\lambda^2-36\lambda+32=0.$$

$$\therefore \lambda=8,2,2.$$

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The Eigen vector corresponding to the Eigen value $\lambda = 8$:-

$$\Rightarrow \begin{bmatrix} 6-8 & -2 & 2 \\ -2 & 3-8 & -1 \\ 2 & -1 & 3-8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -2 & -2 & 2 \\ -2 & -5 & -1 \\ 2 & -1 & -5 \end{bmatrix}$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 + R_1$

$$\Rightarrow \begin{bmatrix} -2 & -2 & 2 \\ 0 & -3 & -3 \\ 0 & -3 & -3 \end{bmatrix}$$

$R_3 \to R_3 - R_2$.

$$\Rightarrow \begin{bmatrix} -2 & -2 & 2 \\ 0 & -3 & -3 \\ 0 & 0 & 0 \end{bmatrix}$$

$\text{Rank} = 2 < n\ (3)$

$\therefore n - r = 3 - 2 = 1$.

$\therefore$ we have to take one arbitrary constant for one variable.

$\therefore$ The eqn's are,

$-2x - 2y - 2z = 0$.

$-3y - 3z = 0$.

Let $z = K$, $y = -K$, $x = 0$.

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = X_1' = \begin{bmatrix} 0 \\ -K \\ K \end{bmatrix} = K \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}. \quad \therefore X_1 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

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Eigen vector corresponding to the Eigen value $\lambda = 2$:

$$\Rightarrow \begin{bmatrix} 6-2 & -2 & 2 \\ -2 & 3-2 & -1 \\ 2 & -1 & 3-2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & -2 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix}$$

$R_2 \to 2R_2 + R_1$, $R_3 \to 2R_3 - R_1$

$$\Rightarrow \begin{bmatrix} 4 & -2 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore \rho(A) = 1 < n(3)$.

$\therefore n - r = 3 - 1 = 2$.

$\therefore$ we have to take two constants for two variables.

$$4x - 2y + 2z = 0.$$

Let $z = k_1$; $y = k_2$

$$\therefore x = \frac{k_2}{2} - \frac{k_1}{2}$$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{k_2}{2} - \frac{k_1}{2} \\ k_2 \\ k_1 \end{bmatrix}$$

$$\therefore X_2 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad X_3 = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$$

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5. (a) Find Rank, Index, signature and Nature of the quadratic form $x^2+2y^2+3z^2+2yz-2zx+2xy$. 5-2

Given quadratic form is,

$$Q = x^2+2y^2+3z^2+2yz-2zx+2xy.$$

quadratic form can be written in the matrix as

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

we write A as, $A = I_3 A I_3$.

$$\Rightarrow \begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow$ $R_2 \to R_2 - R_1$, $R_3 \to R_3 + R_1$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 2 \\ 0 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$C_2 \to C_2 - C_1$, $C_3 \to C_3 + C_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ \phi & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 3 & -2 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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$C_3 \to C_3 - 2C_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 3 & -2 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -1 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

The eigen values are $1, 1, -2$.

Rank(r) = 3.

Index(s) = 2. (Two positive values)

Signature = $2s - r = 2(2) - 3 = 4 - 3 = 1$.

Nature = Indefinite.

(b) If $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the Eigen values of a matrix A. then prove that $1/\lambda_1, 1/\lambda_2, \ldots, 1/\lambda_n$ are the eigen values of A^{-1}.

Proof: Since, A is non singular and product of eigen values is equal to $|A|$, it follows that none of the eigen values of A is zero.

$\therefore$ If λ is the eigen value of A and X is the corresponding eigen vector, $\lambda \neq 0$ and $AX = \lambda X \to (1)$

Premultiplying $A^{-1} \Rightarrow A^{-1}(AX) = A^{-1}(\lambda X)$

$\Rightarrow IX = \lambda A^{-1} X$.

$\therefore X = \lambda^{-1} A^{-1} X$

$\therefore A^{-1} X = \lambda^{-1} X$. $(\because \lambda \neq 0)$

Hence by def it follows that λ^{-1} is an eigen value of A^{-1} and X is the corresponding eigen vector.

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R-20

College Code: 09

Rajeev Gandhi Memorial College of Engineering & Technology
Autonomous
NANDYAL-518501
I B. Tech I-Semester Mid-II *Examinations*
Linear Algebra and Advanced Calculus
Common to CE, EEE, ME, CSE, CSEBS& CSEDS
Date: 10-07-2021

Max. Marks: 20 Time: 2 Hours

Note: 1. Answer first question compulsorily. (5 x 1M = 5 Marks)
2. Answer Any *three* from 2 to 5 questions (3 x 5M = 15 Marks)

Q.1 a) State the Rolle's Theorem.

b) Write Maclaurin's series expansion of e^x.

c) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx\, dy$.

d) Define Beta function.

e) Write any two properties of Gamma function.

Q.2 Prove that $\frac{\pi}{6}+\frac{1}{5\sqrt{3}} < \sin^{-1}\left(\frac{3}{5}\right) < \frac{\pi}{6}+\frac{1}{8}$.

Q.3 a) Find the first and second order partial derivatives of $ax^2+2hxy+by^2$.

b) If $u=\frac{x+y}{1-xy}$ and $\theta = \tan^{-1}x+\tan^{-1}y$ find $\frac{\partial(u,\ \theta)}{\partial(x,\ y)}$.

Q.4 Change the order of integration and evaluate $\int_0^{4a}\int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dy\, dx$.

Q.5 a) Evaluate the triple integral $\int_0^1\int_y^1\int_0^{1-x} x\, dz\, dx\, dy$.

b) Compute i) $\Gamma\left(\frac{11}{2}\right)$ ii) $\Gamma\left(\frac{-1}{2}\right)$

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R-20 ①

College code: 09

Rajeev Gandhi Memorial College of Engineering and Technology.

(Autonomous) Nandyal.

I B.Tech I-Sem Mid-II Examinations.

Linear Algebra and Advanced Calculus

Note:- 1. Answer first question compulsorily.

2. Answer Any three from 2 to 5 questions.

1. (a) State the Rolle's theorem.

If f(x) is a function. which Satisfies the following Conditions.

(i) f(x) is Continuous [a,b].

(ii) f(x) is differentiable (a,b)

(iii) f(a)=f(b). then ∃ a point c such that, $c\in(a,b)$.

i.e $f'(c)=0, \quad c\in(a,b)$.

(b) Write Maclaurins series expansion of e^x.

Maclaurin Series Expansion is

$$f(x) = f(0) + \frac{(x-0)}{1!}f'(0) + \frac{(x-0)^2}{2!}f''(0) + \frac{(x-0)^3}{3!}f'''(0) + \cdots\cdots$$

Given $f(x) = e^x \Rightarrow f(0) = e^0 = 1$

$f'(x) = e^x \Rightarrow f'(0) = e^0 = 1$

$f''(x) = e^x \Rightarrow f''(0) = e^0 = 1$

$f'''(x) = e^x \Rightarrow f'''(0) = e^0 = 1 \cdots$

$\therefore f(x) = e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\cdots$

This is maclaurin series expansion of e^x.

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(C) Evaluate $\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)}\cdot dx\,dy$.

$$\Rightarrow \int_0^{\infty}\int_0^{\infty} e^{-x^2}\cdot e^{-y^2}\cdot dx\,dy.$$

$$\Rightarrow \cancel{\int_0^{\infty}\left[\frac{e^{-x^2}}{-2x}\right]_0^{\infty}}$$

Let us consider,

$x = r\cos\theta,\ y = r\sin\theta.$

$x^2+y^2 = r^2$

Here $r: 0 \to \infty$ then $\theta: 0$ to $\pi/2$.

Now, $\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)}\cdot dx\,dy.$

$$= \int_0^{\pi/2}\int_{r=0}^{\infty} e^{-r^2}\cdot r\,dr\,d\theta$$

Let $r^2 = t$

$2r\,dr = dt.$

$$\Rightarrow \int_0^{\pi/2}\int_0^{\infty}\left(e^{-t}\cdot\frac{dt}{2}\right)d\theta$$

$$= \int_0^{\pi/2}\frac{1}{2}\left(-e^{-t}\right)_0^{\infty}\cdot d\theta.$$

$$= \int_0^{\pi/2}-\frac{1}{2}\left(e^{-r^2}\right)_0^{\infty}\cdot d\theta.$$

$$= \int_0^{\pi/2}-\frac{1}{2}\left(e^{-\infty}-e^{0}\right)d\theta.$$

$$= \int_0^{\pi/2}-\frac{1}{2}(-1)\cdot d\theta.$$

$$\Rightarrow \frac{1}{2}\int_0^{\pi/2}1\cdot d\theta = \frac{1}{2}(\theta)_0^{\pi/2} = \frac{1}{2}\left(\frac{\pi}{2}-0\right) = \frac{\pi}{4}.$$

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(d) Define Beta function:

Beta function: If m, n are positive numbers then the beta function is denoted by $\beta(m,n)$ or $B(m,n)$ and is defined as $B(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$.

(e) Write any two properties of gamma function.

(i) $\Gamma(1) = 1$, (ii) $\Gamma(n) = (n-1)\Gamma(n-1)$, (iii) $\Gamma(n+1) = n!$

2. Prove that $\frac{\pi}{6} + \frac{1}{5\sqrt{3}} < \sin^{-1}\left(\frac{3}{5}\right) < \frac{\pi}{6} + \frac{1}{8}$

Given $\frac{\pi}{6} + \frac{1}{5\sqrt{3}} < \sin^{-1}(3/5) < \frac{\pi}{6} + \frac{1}{8}$.

Let $f(x) = \sin^{-1}(x)$ in $[a,b]$, $0 < a < b < \pi$.

(i) $f(x)$ exists for all values of $x \in [a,b]$.

$\therefore f(x)$ is continuous in $[a,b]$.

(ii) $f'(x) = \frac{1}{\sqrt{1-x^2}}$ which exists for all values of $x \in (a,b)$

$\therefore f(x)$ is differentiable in (a,b).

All the conditions of Lagranges mean value theorem are Satisfied.

$\therefore$ Then there exists $f'(c) = \frac{f(b)-f(a)}{b-a}$.

i.e $\frac{1}{\sqrt{1-c^2}} = \frac{\sin^{-1}(b) - \sin^{-1}(a)}{b-a} \longrightarrow (1)$

$\Rightarrow$ we have $a < c < b$.

$\Rightarrow a^2 < c^2 < b^2$

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$\Rightarrow -a^2 > -c^2 > -b^2$

$\Rightarrow 1-a^2 > 1-c^2 > 1-b^2$

$\Rightarrow \sqrt{1-a^2} > \sqrt{1-c^2} > \sqrt{1-b^2}$

$$\Rightarrow \frac{1}{\sqrt{1-a^2}} < \frac{1}{\sqrt{1-c^2}} < \frac{1}{\sqrt{1-b^2}}$$

by using ①.

$$\Rightarrow \frac{b-a}{\sqrt{1-a^2}} < \sin^{-1}(b) - \sin^{-1}(a) < \frac{b-a}{\sqrt{1-b^2}} \rightarrow ②$$

Taking $a = 1/2$ and $b = 3/5$

$$\Rightarrow \frac{3/5 - 1/2}{\sqrt{1-1/4}} < \sin^{-1}(3/5) - \sin^{-1}(1/2) < \frac{3/5 - \frac{1}{2}}{\sqrt{1-9/25}}$$

$$\Rightarrow \frac{1}{5\sqrt{3}} < \sin^{-1}(3/5) - \frac{\pi}{6} < \frac{1}{8}$$

$$\Rightarrow \frac{\pi}{6} + \frac{1}{5\sqrt{3}} < \sin^{-1}(3/5) < \frac{\pi}{6} + \frac{1}{8}$$

Hence the proof.

3(a) Find the first and second order partial derivatives of $ax^2 + 2hxy + by^2$.

Let given $u = ax^2 + 2hxy + by^2$

$$\frac{\partial u}{\partial x} = 2ax + 2hy. \quad ; \quad \frac{\partial u}{\partial y} = 2hx + 2by.$$

$$\frac{\partial^2 u}{\partial x^2} = 2a \quad ; \quad \frac{\partial^2 u}{\partial y^2} = 2b.$$

$$\frac{\partial u}{\partial x \partial y} = \frac{\partial}{\partial x}(2hx + 2by) = 2h, \quad \frac{\partial u}{\partial y \partial x} = \frac{\partial}{\partial y}(2ax + 2hy) = 2h.$$

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3b) If $u = \frac{x+y}{1-xy}$ and $\theta = \tan^{-1}(x) + \tan^{-1}(y)$. find $\frac{\partial(u,\theta)}{\partial(x,y)}$.

Given $u = \frac{x+y}{1-xy}$, $\theta = \tan^{-1}(x) + \tan^{-1}(y)$.

$$\frac{\partial(u,\theta)}{\partial(x,y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial \theta}{\partial x} & \frac{\partial \theta}{\partial y} \end{vmatrix}$$

let $\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\left(\frac{x+y}{1-xy}\right) = \frac{1+y^2}{(1-xy)^2}$

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\frac{x+y}{1-xy}\right) = \frac{1+x^2}{(1-xy)^2}$$

$$\frac{\partial \theta}{\partial x} = \frac{\partial}{\partial x}\left(\tan^{-1}(x) + \tan^{-1}(y)\right) = \frac{1}{1+x^2}$$

$$\frac{\partial \theta}{\partial y} = \frac{\partial}{\partial y}\left(\tan^{-1}(x) + \tan^{-1}(y)\right) = \frac{1}{1+y^2}$$

Now $\frac{\partial(u,\theta)}{\partial(x,y)} = \begin{vmatrix} \frac{1+y^2}{(1-xy)^2} & \frac{1+x^2}{(1-xy)^2} \\ \frac{1}{1+x^2} & \frac{1}{1+y^2} \end{vmatrix}$

$$= \frac{1}{(1-xy)^2} - \frac{1}{(1-xy)^2}$$

$$= 0.$$

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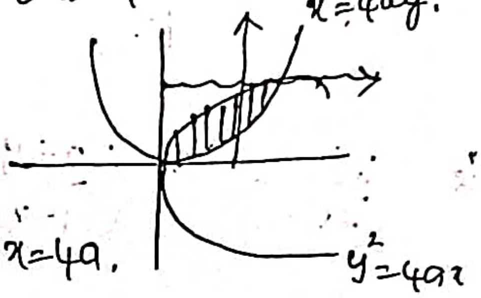
4. Change the order of Integration and evaluate

$$\int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} dy\,dx.$$

Given $\int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} dy\,dx$. Given $y = x^2/4a$ to $2\sqrt{ax}$
$x = 0$ to $4a$.



The region of integration which is bounded by the curve $y = x^2/4a$ and $y = 2\sqrt{ax}$ and $x = 0$ to $x = 4a$, as shown in fig.

Given the order of integration is along the y-axis. By changing into x direction. we have to fix y variable

For fixed y values, x will vary.

$\therefore$ i.e $y: 0$ to $4a$ and $x = \frac{y^2}{4a}$ to $2\sqrt{ay}$.

$$\int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} dy\,dx = \int_0^{4a}\left(\int_{y^2/4a}^{2\sqrt{ay}} dx\right)dy = \int_0^{4a}(x)_{y^2/4a}^{2\sqrt{ay}}\,dy.$$

$$= \int_0^{4a}\left(2\sqrt{ay} - \frac{y^2}{4a}\right)dy = 2\sqrt{a}\left[\frac{(y)^{3/2}}{3/2}\right]_0^{4a} - \frac{1}{4a}\left(\frac{y^3}{3}\right)_0^{4a}$$

$$= \frac{4a^2}{3}\,4\sqrt{4} - \frac{1}{12}\cdot 64a^2$$

$$= \frac{32a^2}{3} - \frac{16a^2}{3} = \frac{16a^2}{3}$$

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5(a) Evaluate the triple integral $\int_0^1 \int_y^1 \int_0^{1-x} x \, dz \, dx \, dy$. 6M

Given $\int_0^1 \int_y^1 \int_0^{1-x} x \cdot dz \, dx \, dy$.

$\Rightarrow \int_{y=0}^1 \int_{x=y}^1 \int_{z=0}^{1-x} x \cdot (z)_0^{1-x} \cdot dx \, dy$.

$\Rightarrow \int_0^1 \int_y^1 x(1-x) \cdot dx \, dy$.

$\Rightarrow \int_0^1 \left(\frac{x^2}{2} - \frac{x^3}{3}\right)_y^1 \cdot dy$.

$\Rightarrow \int_0^1 \left[\frac{1}{2} - \frac{1}{3} - \frac{y^2}{2} + \frac{y^3}{3}\right] \cdot dy$.

$\Rightarrow \left(\frac{1}{6} \cdot y - \frac{1}{2} \cdot \frac{y^3}{3} + \frac{1}{3} \cdot \frac{y^4}{4}\right)_0^1$

$\Rightarrow \left(\frac{1}{6} - \frac{1}{6} + \frac{1}{12}\right)$

$\Rightarrow \frac{1}{12}$

5b) Compute (i) $\Gamma(\frac{11}{2})$ (ii) $\Gamma(-\frac{1}{2})$.

We know that $\Gamma(n) = (n-1)\,\Gamma(n-1)$ (or)

$\Gamma(n+1) = n\,\Gamma(n)$.

(i) let $\Gamma(\frac{11}{2}) = \Gamma(\frac{9}{2}+1) = \frac{9}{2}\Gamma(\frac{9}{2})$

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$$= \frac{9}{2}\Gamma\left(\frac{7}{2}+1\right)$$

$$= \frac{9}{2}\cdot\frac{7}{2}\cdot\Gamma\left(\frac{7}{2}\right)$$

$$= \frac{9}{2}\cdot\frac{7}{2}\cdot\Gamma\left(\frac{5}{2}+1\right)$$

$$= \frac{9}{2}\cdot\frac{7}{2}\cdot\frac{5}{2}\cdot\Gamma(5/2)$$

$$= \frac{9}{2}\cdot\frac{7}{2}\cdot\frac{5}{2}\cdot\Gamma\left(\frac{3}{2}+1\right)$$

$$= \frac{9}{2}\cdot\frac{7}{2}\cdot\frac{5}{2}\cdot\frac{3}{2}\cdot\Gamma\left(\frac{3}{2}\right)$$

$$= \frac{9}{2}\cdot\frac{7}{2}\cdot\frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\Gamma\left(\frac{1}{2}\right)$$

$$\boxed{\Gamma\left(\frac{11}{2}\right) = \frac{9}{2}\cdot\frac{7}{2}\cdot\frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\cdot\sqrt{\pi}}$$

(ii) For $\Gamma(-1/2)$, we know that

$$\Gamma(1-n)\cdot\Gamma(n) = \frac{\pi}{\sin n\pi} \longrightarrow (1)$$

let $n=-1/2 \Rightarrow \Gamma\left(1+\frac{1}{2}\right)\Gamma\left(-\frac{1}{2}\right) = \frac{\pi}{\sin\left(-\frac{\pi}{2}\right)}$

$$= \Gamma\left(-\frac{1}{2}\right) = \frac{\pi}{-\sin\left(\frac{\pi}{2}\right)}\cdot\frac{1}{\frac{1}{2}\Gamma\left(\frac{1}{2}\right)}$$

$$= \frac{-2\pi}{\sqrt{\pi}}$$

$$\boxed{\Gamma\left(-\frac{1}{2}\right) = -2\sqrt{\pi}}$$

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End Examination Paper with scheme

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Code: A0001201R0721

3

RGM COLLEGE OF ENGINEERING & TECHNOLOGY (AUTONOMOUS)

11th July-2021

I B.Tech I Semester (R20) End Examinations (Regular)

LINEAR ALGEBRA AND ADVANCED CALCULUS

Common to CE, ME, EEE, CSE, CSEDS & CSEBS

Time: 3 Hrs **Total Marks: 70**

Note 1:Answer Question No.1 (Compulsory) and 4 from the remaining

2:All Questions Carry Equal Marks

1a Evaluate $\int_0^{\pi}\int_0^{a\sin\theta} r\,dr\,d\theta$

b Find the rank of the matrix $A=\begin{bmatrix}-1 & 0 & 6\\ 3 & 6 & 1\\ -5 & 1 & 3\end{bmatrix}$.

c State the definition of Eigen value and Eigen vector.

d Verify Roll's theorem for the function $f(x)=x^2-2x-3$ in $[-1,3]$.

e Obtain the quadratic form corresponding to the matrix

$$\begin{bmatrix}1 & 2 & 3\\ 2 & 0 & 3\\ 3 & 3 & 1\end{bmatrix}$$

f Evaluate $\int_0^{\infty} x^6 e^{-2x}\,dx$

g Evaluate $\int_0^{\pi}\int_0^{\sin\theta} r\,dr\,d\theta$.

2 a) Evaluate $\iint_R (x+y)^2\,dxdy$, where R is the region bounded by $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. (7)

b) Evaluate $\iint_R r^2\sin\theta\,drd\theta$, where R is the semi-circle $r=2a\cos\theta$ above the initial line. (7)

Find the Eigen values and Eigen vectors of $A=\begin{bmatrix}2 & 3 & 4\\ 0 & 4 & 2\\ 0 & 0 & 3\end{bmatrix}$

a) Show that $u=2x-y+3z, v=2x-y-z, w=2x-y+z$, are functionally dependent and find the relation. (7)

b) If $V=f(2x-3y, 3y-4z, 4z-2x)$, then prove that $6\frac{\partial V}{\partial x}+4\frac{\partial V}{\partial y}+3\frac{\partial V}{\partial z}=0$. (7)

a) Show that $\Gamma(n)=\int_0^1 \log\left(\frac{1}{x}\right)^{n-1}dx,\ n>0$. (7)

b) Show that $a^m b^n\int_0^{\infty}\frac{x^{m-1}}{(ax+b)^{m+n}}dx=\beta(m,n)$. (7)

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Code: A0001201R0721

6

a) Find rank of a matrix $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ by reducing it into Echelonform. (7)

b) Solve the following system of equations, (7)
$3x + 2y + z = 3;\ 2x + y + z = 0;\ 6x + 2y + 4z = 6.$

7 Reduce the quadratic form $x^2 + 2y^2 - 7z^2 + 8zx - 4xy$ in to canonical form and find the Rank, Index, Signature and Nature.

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RGM COLLEGE OF ENGINEERING & TECHNOLOGY (AUTONOMOU

I B. Tech I Semester (R20) End Examinations (Regular,

Linear Algebra and Advanced Calculus

Common to CE, ME, EEE, CSE, CSE.DS & CSE BS.

Scheme of Evaluation.

Code: A0001201R0721

Max Marks : 70

1)a) $\int_0^{\pi}\int_0^{a\sin\theta} r\,dr\,d\theta = \int_0^{\pi}\left[\int_0^{a\sin\theta} r\,dr\right]d\theta = \int_0^{\pi}\left[\frac{r^2}{2}\right]_0^{a\sin\theta} d\theta$

$= \frac{a^2}{2}\int_0^{\pi}\sin^2\theta\,d\theta = \frac{a^2}{2}\int_0^{\pi}\frac{1-\cos2\theta}{2}\,d\theta$ — 1M

$= \frac{a^2}{4}\int_0^{\pi}(1-\cos2\theta)\,d\theta = \frac{a^2}{4}\left(\theta - \frac{\sin2\theta}{2}\right)_0^{\pi}$

$= \frac{a^2}{4}\left[(\pi-0)-(0-0)\right] = \frac{a^2\pi}{4}.$ — 1M

b) Given that $A = \begin{bmatrix} -1 & 0 & 6 \\ 3 & 6 & 1 \\ -5 & 1 & 3 \end{bmatrix}$

$\sim \begin{bmatrix} -1 & 0 & 6 \\ 0 & 6 & 19 \\ 0 & 1 & -27 \end{bmatrix}$ $R_2 \to R_2 + 3R_1$, $R_3 \to R_3 - 5R_1$

$\sim \begin{bmatrix} -1 & 0 & 6 \\ 0 & 6 & 19 \\ 0 & 0 & -181 \end{bmatrix}$ $R_3 \to 6R_3 - R_2$ — 1M

$\therefore \rho(A) = 3$ — 1M

c) Eigen vector: Let $A = [a_{ij}]$ be $n\times n$ matrix. A non-ze vector X is said to be a "Eigen vector" of A if there exist a scalar λ such that $AX = \lambda X$. — 1M

Eigen value: If $Ax = \lambda x$, $(x \neq 0)$ we say that x is Eigen vector of A corresponding to the eigen value λ of A. (or) If $Ax = \lambda x$ $(x\neq 0)$, x is a vector then λ is the Eigen value of A. — 1M

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d) Given $f(x) = x^2 - 2x - 3$ in $[-1, 3]$ Then $f'(x) = 2x - 2$.
This is a polynomial in x. so it is continuous and differentiable in $[-1, 3]$ and $(-1, 3)$ respectively.

$$f(-1) = (-1)^2 - 2(-1) - 3 = 1 + 2 - 3 = 0$$
$$f(3) = 3^2 - 2(3) - 3 = 9 - 6 - 3 = 0.$$
$$f(-1) = f(3).$$

$\therefore$ All the conditions of Rolle's theorem are verified.
So there exist least one point $c \in (-1, 3)$ such that

$$f'(c) = 0 \quad \text{i.e} \quad 2c - 2 = 0 \Rightarrow c = 1.$$

Clearly $c = 1 \in (-1, 3)$.
Hence Rolle's theorem is verified.

e) Given that $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 3 \\ 3 & 3 & 1 \end{bmatrix}$ (say) let $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

Then Quadratic form corresponding to A is — 1M

$$Q = X^T A X$$
$$= \begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 3 \\ 3 & 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$
$$= \begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} x + 2y + 3z \\ 2x + 3z \\ 3x + 3y + z \end{bmatrix}$$
$$= x(x + 2y + 3z) + y(2x + 3z) + z(3x + 3y + z)$$
$$= x^2 + 2xy + 3zx + 2xy + 3yz + 3zx + 3yz + z^2$$
$$= x^2 + z^2 + 4xy + 6zx + 6yz. \quad \text{— 1M}$$

f) $\int_0^\infty x^6 e^{-2x} dx$

let $2x = y$ so that $dx = \frac{1}{2} dy$

$$\therefore \int_0^\infty x^6 e^{-2x} dx = \int_0^\infty \left(\frac{y}{2}\right)^6 e^{-y} \cdot \frac{1}{2} dy = \frac{1}{2^7} \int_0^\infty e^{-y} y^6 dy \quad \text{— 1M}$$
$$= \frac{1}{2^7} \int_0^\infty e^{-y} y^{7-1} dy = \frac{1}{2^7} \Gamma(7) = \frac{1}{2^7} \times 6! = \frac{45}{8} \quad \text{— 1M}$$

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9) $\int_0^{\pi}\int_0^{\sin\theta} r\,dr\,d\theta = \int_0^{\pi}\left[\int_0^{\sin\theta} r\,dr\right]d\theta = \int_0^{\pi}\left[\frac{r^2}{2}\right]_0^{\sin\theta} d\theta$ b)

$= \frac{1}{2}\int_0^{\pi}\sin^2\theta\,d\theta = \frac{1}{2}\int_0^{\pi}\left(\frac{1-\cos 2\theta}{2}\right)d\theta$ — 1M

$= \frac{1}{4}\int_0^{\pi}(1-\cos 2\theta)\,d\theta = \frac{1}{4}\left(\theta - \frac{\sin 2\theta}{2}\right)_0^{\pi}$

$= \frac{1}{4}\left[(\pi - 0)-(0-0)\right] = \pi/4.$ — 1M

2)

a) The equation of the ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \Rightarrow y = \pm\frac{b}{a}\sqrt{a^2-x^2}$

we have the region of integration is $-a\le x\le a$,

$-\frac{b}{a}\sqrt{a^2-x^2}\le y\le \frac{b}{a}\sqrt{a^2-x^2}$ — 1M

$\therefore \iint_R (x^2+y^2+2xy)\,dx\,dy = \int_{x=-a}^{a}\int_{y=-\frac{b}{a}\sqrt{a^2-x^2}}^{+\frac{b}{a}\sqrt{a^2-x^2}} (x^2+y^2+2xy)\,dx\,dy.$

$\therefore I = \iint_R (x+y)^2\,dx\,dy = \int_{x=-a}^{a}\int_{y=-\frac{b}{a}\sqrt{a^2-x^2}}^{\frac{b}{a}\sqrt{a^2-x^2}}(x^2+y^2)\,dx\,dy + \int_{x=-a}^{a}\int_{y=-\frac{b}{a}\sqrt{a^2-x^2}}^{\frac{b}{a}\sqrt{a^2-x^2}} 2xy\,dx$

$= 2\int_{x=-a}^{a}\int_{y=0}^{\frac{b}{a}\sqrt{a^2-x^2}}(x^2+y^2)\,dy\,dx + 0$ — 2M

[$\because (x^2+y^2)$ is even function and xy is an odd function]

$= 2\int_{x=-a}^{a}\left[x^2y+\frac{y^3}{3}\right]_{y=0}^{\frac{b}{a}\sqrt{a^2-x^2}}dx.$

$= 2\int_{x=-a}^{a}\left[x^2\frac{b}{a}\sqrt{a^2-x^2}+\frac{1}{3}\frac{b^3}{a^3}(a^2-x^2)^{3/2}\right]dx$

$= 4\int_{x=0}^{a}\left[\frac{b}{a}x^2\sqrt{a^2-x^2}+\frac{b^3}{3a^3}(a^2-x^2)^{3/2}\right]dx.$ — 2M

Put $x = a\sin\theta$ so that $dx = a\cos\theta\,d\theta$. Also $x=0 \Rightarrow \theta=0$
$x=a \Rightarrow \theta=\pi/2$

$\therefore I = 4\int_0^{\pi/2}\left[\frac{b}{a}a^2\sin^2\theta\, a\cos\theta + \frac{b^3}{3a^3}a^3\cos^3\theta\right]a\cos\theta\,d\theta$

$= 4\int_0^{\pi/2}\left[a^3b\sin^2\theta\cos^2\theta + \frac{ab^3}{3}\cos^4\theta\right]d\theta.$

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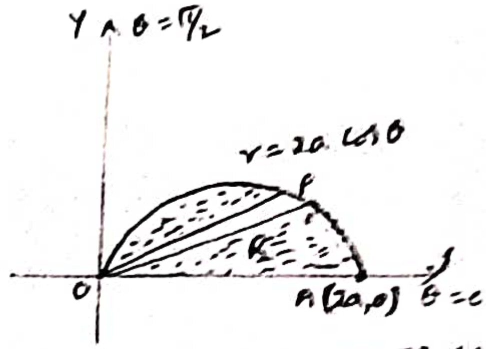
$$= 4\left[a^3 b \, \frac{1}{4}\cdot\frac{1}{2}\cdot\frac{\pi}{2} + \frac{ab^3}{3}\,\frac{3}{4}\cdot\frac{1}{2}\cdot\frac{1}{2}\right]$$ 68

$$= \frac{\pi}{4}\left(a^3 b + ab^3\right) = \frac{\pi}{4}\, ab\left(a^2+b^2\right).$$ — 2M

2) b) The semi-circle $r = 2a\cos\theta$ passes through the pole. The region of integration R is shown in the figure as shaded region. Here r varies from 0 to $2a\cos\theta$ while θ varies from 0 to $\frac{\pi}{2}$. — 1M



— 2M

$$\therefore \iint_R r^2 \sin\theta \, dr\, d\theta$$

$$= \int_{\theta=0}^{\pi/2} \int_{r=0}^{2a\cos\theta} r^2 \sin\theta \, dr\, d\theta$$

$$= \int_{\theta=0}^{\pi/2} \left\{ \int_{r=0}^{2a\cos\theta} r^2\, dr \right\} d\theta$$ — 1M

$$= \int_{\theta=0}^{\pi/2} \left(\frac{r^3}{3}\right)_{r=0}^{2a\cos\theta} d\theta = \frac{1}{3}\int_{\theta=0}^{\pi/2} \sin\theta\left(8a^3\cos^3\theta - 0\right) d\theta$$

$$= \frac{8a^3}{3}\int_{\theta=0}^{\pi/2} \sin\theta\cos^3\theta\, d\theta = 8\frac{a^3}{3}\left(-\frac{\cos^4\theta}{4}\right)_{\theta=0}^{\pi/2}$$

$$= -\frac{2a^3}{3}\left[\cos^4\frac{\pi}{2} - \cos^4 0\right] = -\frac{2a^3}{3}(0-1) = \frac{2a^3}{3}.$$ — 2M

3) Given $A = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 4 & 2 \\ 0 & 0 & 3 \end{bmatrix}$

Characteristic eqn of A is $|A - \lambda I| = 0$

i.e $\begin{vmatrix} 2-\lambda & 3 & 4 \\ 0 & 4-\lambda & 2 \\ 0 & 0 & 3-\lambda \end{vmatrix} = 0$ — 2M

$$\Rightarrow (2-\lambda)\begin{vmatrix} 4-\lambda & 2 \\ 0 & 3-\lambda \end{vmatrix} - (0)\begin{vmatrix} 3 & 4 \\ 0 & 3-\lambda \end{vmatrix} + (0)\begin{vmatrix} 3 & 4 \\ 4-\lambda & 2 \end{vmatrix} = 0$$

$$\Rightarrow (2-\lambda)\left[(4-\lambda)(3-\lambda) - 0\right] - 0 + 0 = 0.$$

$$\Rightarrow (2-\lambda)(4-\lambda)(3-\lambda) = 0$$

$$\Rightarrow \lambda = 2, 3, 4.$$ — 2M

$\therefore$ Eigen values of A are $2, 3$ and 4.

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Eigen vector corresponding to $\lambda = 2$: $[A-\lambda I]X = 0$

$$\begin{bmatrix} 2-2 & 3 & 4 \\ 0 & 4-2 & 2 \\ 0 & 0 & 3-2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 0 & 3 & 4 \\ 0 & 2 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$\Rightarrow 3y + 4z = 0$ —— (1), $2y + 2z = 0$ —— (2)

$\Rightarrow y = z = 0$

Let $x = \alpha$ then $X = \begin{bmatrix} \alpha \\ 0 \\ 0 \end{bmatrix} = \alpha\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$\therefore$ Eigen vector corresponding to $\lambda = 2$ is $X_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$. —— 3M

Eigen vector corresponding to $\lambda = 3$: $[A-\lambda I]X = 0$.

$$\begin{bmatrix} 2-3 & 3 & 4 \\ 0 & 4-3 & 2 \\ 0 & 0 & 3-3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0.$$

$$\Rightarrow \begin{bmatrix} -1 & 3 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$\Rightarrow -x + 3y + 4z = 0$ —— (1) $\quad y + 2z = 0$ —— (2)

Let $z = \alpha$ then $y = -2\alpha$

(1) $\Rightarrow -x + 3(-2\alpha) + 4(\alpha) = 0$

$\Rightarrow -x - 2\alpha = 0 \Rightarrow x = -2\alpha$.

$\therefore X = \begin{bmatrix} -2\alpha \\ -2\alpha \\ \alpha \end{bmatrix} = \alpha\begin{bmatrix} -2 \\ -2 \\ 1 \end{bmatrix}$.

$\therefore$ Eigen vector corresponding to $\lambda = 3$ is $X_2 = \begin{bmatrix} -2 \\ -2 \\ 1 \end{bmatrix}$. —— 3M

Eigen vector corresponding to $\lambda = 4$: $[A-\lambda I]X = 0$

$$\begin{bmatrix} 2-4 & 3 & 4 \\ 0 & 4-4 & 2 \\ 0 & 0 & 3-4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} -2 & 3 & 4 \\ 0 & 0 & 2 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0.$$

$R_1 \to \frac{R_1}{-2}$ $\quad \begin{bmatrix} 1 & -3/2 & -2 \\ 0 & 0 & 2 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

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$R_2 \to \frac{R_2}{2}$ $\begin{bmatrix} 1 & -3/2 & -2 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$R_1 \to R_1 + 2R_2$ $\begin{bmatrix} 1 & -3/2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$R_3 \to R_3 + R_2$ $\begin{bmatrix} 1 & -3/2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$.

$x - \frac{3}{2}y = 0$, $z = 0$ Let $y = \alpha$ then

$x - \frac{3}{2}\alpha = 0 \Rightarrow x = \frac{3}{2}\alpha$

$\therefore X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3/2\,\alpha \\ \alpha \\ 0 \end{bmatrix} = \alpha\begin{bmatrix} 3/2 \\ 1 \\ 0 \end{bmatrix} = \alpha\begin{bmatrix} 1.5 \\ 1 \\ 0 \end{bmatrix}$. — 3M

$\therefore$ Eigen vector corresponding to $\lambda = 4$ is $X_3 = \begin{bmatrix} 1.5 \\ 1 \\ 0 \end{bmatrix}$.

Eigen values are 2, 3, 4 and corresponding eigen vectors are $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1.5 \\ 1 \\ 0 \end{bmatrix}$. — 1M

4) a) Given that $u = 2x - y + 3z$, $v = 2x - y - z$, $w = 2x - y + z$.

$\frac{\partial u}{\partial x} = 2$, $\frac{\partial u}{\partial y} = -1$, $\frac{\partial u}{\partial z} = 3$

$\frac{\partial v}{\partial x} = 2$, $\frac{\partial v}{\partial y} = -1$, $\frac{\partial v}{\partial z} = -1$.

$\frac{\partial w}{\partial x} = 2$, $\frac{\partial w}{\partial y} = -1$, $\frac{\partial w}{\partial z} = 1$. — 2M

$\therefore J\left(\frac{u, v, w}{x, y, z}\right) = \frac{\partial(u, v, w)}{\partial(x, y, z)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix}$ — 1M

$= \begin{vmatrix} 2 & -1 & 3 \\ 2 & -1 & -1 \\ 2 & -1 & 1 \end{vmatrix}$

$= \begin{vmatrix} 1 & 1 & 3 \\ 1 & 1 & -1 \\ 1 & 1 & 1 \end{vmatrix}$ $C_1 \to \frac{C_1}{2}$, $C_2 \to C_2/-1$

$= 0$ $[\because C_1 = C_2]$. — 2M

$\therefore$ u, v, w are functionally related. (i.e dependent).

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$$u+v = (2x-y+3z)+(2x-y-z)$$
$$= 4x-2y+2z$$
$$= 2(2x-y+z) = 2w$$

$\therefore$ $\boxed{u+v=2w}$ is the relation between u, v and w.

— 2M

4) b) Given that $v = f(2x-3y, 3y-4z, 4z-2x)$.

Let $r = 2x-3y$, $s = 3y-4z$, $t = 4z-2x$. — 1M

Then $v = f(r, s, t)$

$$\frac{\partial r}{\partial x} = 2, \quad \frac{\partial r}{\partial y} = -3, \quad \frac{\partial r}{\partial z} = 0$$

$$\frac{\partial s}{\partial x} = 0, \quad \frac{\partial s}{\partial y} = 3, \quad \frac{\partial s}{\partial z} = -4$$

$$\frac{\partial t}{\partial x} = -2, \quad \frac{\partial t}{\partial y} = 0, \quad \frac{\partial t}{\partial z} = 4.$$ — 3M

By the chain rule of partial differentiation

$$\frac{\partial v}{\partial x} = \frac{\partial v}{\partial r}\cdot\frac{\partial r}{\partial x} + \frac{\partial v}{\partial s}\cdot\frac{\partial s}{\partial x} + \frac{\partial v}{\partial t}\cdot\frac{\partial t}{\partial x}$$
$$= 2\frac{\partial v}{\partial r} - 2\frac{\partial v}{\partial t} \quad \text{--- (1)}$$

$$\frac{\partial v}{\partial y} = \frac{\partial v}{\partial r}\cdot\frac{\partial r}{\partial y} + \frac{\partial v}{\partial s}\cdot\frac{\partial s}{\partial y} + \frac{\partial v}{\partial t}\cdot\frac{\partial t}{\partial y}$$
$$= -3\frac{\partial v}{\partial r} + 3\frac{\partial v}{\partial s} \quad \text{--- (2)}$$

$$\frac{\partial v}{\partial z} = \frac{\partial v}{\partial r}\cdot\frac{\partial r}{\partial z} + \frac{\partial v}{\partial s}\cdot\frac{\partial s}{\partial z} + \frac{\partial v}{\partial t}\cdot\frac{\partial t}{\partial z}.$$
$$= -4\frac{\partial v}{\partial s} + 4\frac{\partial v}{\partial t} \quad \text{--- (3)}$$ — 2M

$$\therefore 6\frac{\partial v}{\partial x} + 4\frac{\partial v}{\partial y} + 3\frac{\partial v}{\partial z} = 6\left[2\frac{\partial v}{\partial r} - 2\frac{\partial v}{\partial t}\right] + 4\left[-3\frac{\partial v}{\partial r} + 3\frac{\partial v}{\partial s}\right] + 3\left[-4\frac{\partial v}{\partial s} + 4\frac{\partial v}{\partial t}\right]$$

$$= 12\frac{\partial v}{\partial r} - 12\frac{\partial v}{\partial t} + (-12)\frac{\partial v}{\partial r} + 12\frac{\partial v}{\partial s} - 12\frac{\partial v}{\partial s} + 12\frac{\partial v}{\partial t}$$
$$= 0$$

— 1M

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5) a) we have $\Gamma(n) = \int_0^\infty e^{-x} x^{n-1} dx$ — ① ——3M 7M

Taking $x = \log \frac{1}{y} = -\log y$

or $y = e^{-x}$ so that $dy = -e^{-x} dx$ or $dx = -\frac{1}{y} dy$ ——3M

Then ① becomes

$$\Gamma(n) = -\int_1^0 \left(\log \frac{1}{y}\right)^{n-1} y \cdot \frac{1}{y} dy = \int_0^1 \left(\log \frac{1}{y}\right)^{n-1} dy$$

or $\Gamma(n) = \int_0^1 \left(\log \frac{1}{x}\right)^{n-1} dx$. ——2M

5) b) we have to prove that $a^m b^n \int_0^\infty \frac{x^{m-1}}{(ax+b)^{m+n}} dx = B(m,n)$

$$a^m b^n \int_0^\infty \frac{x^{m-1}}{(ax+b)^{m+n}} dx = a^m b^n \int_0^\infty \frac{x^{m-1}}{b^{m+n}\left(\frac{ax}{b}+1\right)^{m+n}} dx$$ ——1M

put $\frac{ax}{b} = t$ then $dx = \frac{b}{a} dt$ and $x = \frac{bt}{a}$ ——2M

$$\therefore a^m b^n \int_0^\infty \frac{x^{m-1}}{(ax+b)^{m+n}} dx = \frac{a^m b^n}{b^{m+n}} \int_0^\infty \frac{\frac{b^{m-1}}{a^{m-1}} t^{m-1} \frac{b}{a} dt}{(t+1)^{m+n}}$$

$$= \int_0^\infty \frac{t^{m-1} dt}{(1+t)^{m+n}}$$ ——2M

$$= \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx.$$

$$= B(m,n).$$

$$a^m b^n \int_0^\infty \frac{x^{m-1}}{(ax+b)^{m+n}} dx = B(m,n).$$ ——2M

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6) a) Let $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$

$$\sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix} \quad R_1 \leftrightarrow R_2$$ — 2M

$$\sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix} \quad \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \\ R_4 \to R_4 - 6R_1 \end{array}$$

$$\sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 33 & 22 \end{bmatrix} \quad \begin{array}{l} R_3 \to 5R_3 - 4R_2 \\ R_4 \to 5R_4 - 9R_2 \end{array}$$ — 2M

$$\sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 3 & 2 \end{bmatrix} \quad \begin{array}{l} R_3 \to R_3/11 \\ R_4 \to R_4/11 \end{array}$$

$$\sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_4 \to R_4 - R_3$$ — 2M

— 1M

$\therefore \ \rho(A) = 3.$

6) b) The given system of eq$_n$ is

$$3x + 2y + z = 3$$
$$2x + y + z = 0$$
$$6x + 2y + 4z = 6$$

This can be written as $AX = B$ — 2M

where $A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 1 \\ 6 & 2 & 4 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 3 \\ 0 \\ 6 \end{bmatrix}$

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$\therefore$ Augmented matrix $[A/B] = \begin{bmatrix} 3 & 2 & 1 & 3 \\ 2 & 1 & 1 & 0 \\ 6 & 2 & 4 & 6 \end{bmatrix}$ 74.

$$\sim \begin{bmatrix} 3 & 2 & 1 & 3 \\ 0 & -1 & 1 & -6 \\ 0 & -2 & 2 & 0 \end{bmatrix} \begin{matrix} R_2 = 3R_2 - 2R_1 \\ R_3 = R_3 - 2R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 3 & 2 & 1 & 3 \\ 0 & -1 & 1 & -6 \\ 0 & 0 & 0 & 12 \end{bmatrix} R_3 = R_3 - 2R_2$$

— 3M

$\rho(A/B) = 3, \quad \rho(A) = 2.$

$\therefore \rho(A/B) \neq \rho(A)$

$\therefore$ The given system does not have solution.

i.e The given system is inconstistant. — 2M

7) Given quadratic form is $Q = x^2 + 2y^2 - 7z^2 + 8zx - 4xy$

	x	y	z
x	1	-2	4
y	-2	2	0
z	4	0	-7

$\therefore$ Matrix of the given Q.F is $A = \begin{bmatrix} 1 & -2 & 4 \\ -2 & 2 & 0 \\ 4 & 0 & -7 \end{bmatrix}$ — 2M

we consider $A_{3\times 3} = I_3 A I_3$

i.e $\begin{bmatrix} 1 & -2 & 4 \\ -2 & 2 & 0 \\ 4 & 0 & -7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ — 2M

Applying $R_2 \to R_2 + 2R_1$, $R_3 \to R_3 - 4R_1$

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$$\begin{bmatrix} 1 & -2 & 4 \\ 0 & -2 & 8 \\ 0 & 8 & -23 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Applying $C_2 \to C_2 + 2C_1$, $C_3 \to C_3 - 4C_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 8 \\ 0 & 8 & -23 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 2 & -4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{— 2M}$$

Applying $R_3 \to R_3 + 4R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 8 \\ 0 & 0 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 4 & 4 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 2 & -4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Applying $C_3 \to C_3 + 4C_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 4 & 4 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{— 2M}$$

$\therefore$ Applying $R_2 \to \frac{R_2}{\sqrt{2}}$, $R_3 \to \frac{R_3}{\sqrt{9}}$, $C_2 \to \frac{C_2}{\sqrt{2}}$, $C_3 \to \frac{C_2}{\sqrt{9}}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \sqrt{2} & 1/\sqrt{2} & 0 \\ 4/3 & 4/3 & 1/3 \end{bmatrix} A \begin{bmatrix} 1 & \sqrt{2} & 4/3 \\ 0 & 1/\sqrt{2} & 4/3 \\ 0 & 0 & 1/3 \end{bmatrix} \quad \text{— 2M}$$

$\therefore$ Normal form $= y_1^2 - y_2^2 + y_3^2$

Rank $(r) = 3$, Index $(s) = 2$ — 2M

Signature $= 2s - r = 2(2) - 3 = 1$

Nature is Indefinite. — 2M

—x—

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Co’s and Po’s attainment

CO's and PO's attainment

Subject:LA&AC

Table 8.1: Evaluation of Question Paper Weightage from Internal Question Papers

Linear Algebra and Advanced Calculus MID Questions Weightage to CO

Academic Year 2020-2021 (2020 Batch)

MID –I

Q.No.	CO 1	CO 2	CO 3	CO 4	CO 5	Total
1 a)			1			1
b)	1					1
c)			1			1
d)					1	1
e)					1	1
2 a)		3				3
b)				2		2
3 a)			3			3
b)	2					2
4 a)					3	3
b)			2			2
5 a)	3					3
b)		2				2
Total	7	5	9	2	7	25

MID –II

Q.No.	CO 1	CO 2	CO 3	CO 4	CO 5	Total
1 a)	1					1
b)			1			1
c)		1				1
d)					1	1
e)				1		1
2 a)	3					3
b)		2				2
3 a)					3	3
b)		2				2
4 a)			3			3
b)	2					2
5 a)					2	2
b)			3			3
Total	7	6	8	2	7	25
Internal	12	10	14	3	11	50

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Table-8.2: Evaluation of Question Paper Weightage from End Exam Question Papers

Linear Algebra and Advanced Calculus, Academic Year 2020-2021 (2020 Batch)						
END EXAM						
Q.No.	**CO 1**	**CO 2**	**CO 3**	**CO 4**	**CO 5**	**Total**
1 a)			**2**			**2**
b)		**2**				**2**
c)	**2**					**2**
d)					**2**	**2**
e)				**2**		**2**
f)					**2**	**2**
g)					**2**	**2**
2 a)					**7**	**7**
b)					**7**	**7**
c)						**0**
3 a)		**7**				**7**
b)		**7**				**7**
c)						**0**
4 a)			**7**			**7**
b)			**7**			**7**
c)						**0**
5 a)			**7**			**7**
b)			**7**			**7**
c)						**0**
6 a)		**7**				**7**
b)	**7**					**7**
c)						**0**
7 a)				**14**		**14**
b)						**0**
c)						**0**
END EXAM	9	23	30	16	20	98

Table-8.3 Question Paper Weightages in CO_i in %.

Category/CO's	CO1	CO2	CO3	CO4	CO5
End Semester Exam (QEM)	9.183673	23.46939	30.6122449	16.32653	20.40816
Internal Exam (QIM)	24	20	28	6	22
Assignment (QA)	22.5	20	27.5	0	30
Average ($AQCO_i$)	13.47857	22.42857	29.7785714	12.62857	21.68571

For theory subject the total marks is 100, evaluated as 70 marks for End semester examination, 20 marks from two internal tests and 10 marks from two assignments. For evaluation CO_i the weights of marks considered are shown in Table 8.4.

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Table - 8. 4. 1: CO attainment Calculation for the course Linear Algebra and Advanced Calculus:

Calculation of CO Attainment of Subject								
A: Assignment Marks, IM: Internal Marks, FE: Final Exam Marks								
Reg.NO:	A	IM	FE	CO-1	CO-2	CO-3	CO-4	CO-5
19091A0553	8	5	27	40.65	39.85	39.85	37.28	41.55
20091A0501	10	18.75	60	90.96	88.42	88.54	86.48	89.32
20091A0502	10	16.75	55	83.99	81.41	81.52	79.06	82.59
20091A0503	10	16	53	81.29	78.64	78.76	76.12	79.94
20091A0504	10	18.25	59	89.39	86.93	87.05	84.95	87.87
20091A0505	10	10.75	40	63.09	60.36	60.46	56.82	62.38
20091A0506	10	14.75	32	64.76	55.56	56.00	48.38	58.91
20091A0507	10	19	52	85.96	80.27	80.56	76.25	82.05
20091A0508	10	19	63	93.45	91.78	91.86	90.48	92.40
20091A0509	9.5	5.75	6	30.19	19.88	20.35	10.49	24.62
20091A0510	10	17	55	84.44	81.63	81.76	79.18	82.84
20091A0511	10	18.25	56	87.34	83.79	83.96	81.07	85.05
20091A0512	10	18.5	52	85.06	79.83	80.09	76.02	81.54
20091A0513	10	16.5	54	82.87	80.14	80.26	77.65	81.39
20091A0514	10	19	58	90.04	86.55	86.72	84.01	87.69
20091A0515	10	16.5	45	76.73	70.72	71.01	66.02	72.92
20091A0516	10	19	60	91.41	88.64	88.78	86.60	89.57
20091A0517	10	19	52	85.96	80.27	80.56	76.25	82.05
20091A0518	10	18.25	57	88.03	84.84	84.99	82.36	85.99
20091A0519	10	9	41	60.65	59.85	59.85	57.28	61.55
20091A0520	10	10.5	0	35.39	18.28	19.11	4.99	24.49
20091A0521	10	18.75	59	90.28	87.37	87.52	85.19	88.38
20091A0522	10	18	51	83.49	78.33	78.59	74.49	80.09
20091A0523	10	18.75	54	86.87	82.14	82.38	78.72	83.67
20091A0524	10	18	58	88.26	85.66	85.78	83.54	86.68
20091A0525	10	18.5	62	91.88	90.29	90.37	88.94	90.95
20091A0526	10	17.75	59	88.50	86.48	86.58	84.71	87.37
20091A0527	10	8.5	37	57.04	55.21	55.26	51.87	57.28
20091A0528	10	19	63	93.45	91.78	91.86	90.48	92.40
20091A0529	10	18.5	62	91.88	90.29	90.37	88.94	90.95
20091A0530	10	18.25	58	88.71	85.88	86.02	83.65	86.93
20091A0531	10	18.75	55	87.55	83.19	83.40	80.01	84.62
20091A0532	10	18	52	84.17	79.38	79.62	75.78	81.03
20091A0533	10	19	59	90.72	87.60	87.75	85.30	88.63
20091A0534	10	19	61	92.09	89.69	89.81	87.89	90.52
20091A0535	10	14	45	72.28	68.49	68.66	64.83	70.39
20091A0536	10	16.75	42	75.14	67.80	68.16	62.26	70.35

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20091A0537	10	8	25	47.97	42.21	42.46	36.12	45.48
20091A0538	10	15.5	41	72.23	65.64	65.96	60.37	68.14
20091A0539	9	11.75	43	65.24	63.50	63.56	61.17	64.84
20091A0540	10	11	27	54.68	46.98	47.33	40.13	50.40
20091A0541	10	18	63	91.67	90.89	90.92	90.00	91.38
20091A0542	10	12.25	52	73.94	74.25	74.21	73.05	75.20
20091A0543	10	16.5	50	80.14	75.95	76.15	72.48	77.63
20091A0544	10	19	65	94.81	93.88	93.92	93.06	94.28
20091A0545	10	17.5	58	87.37	85.21	85.31	83.30	86.17
20091A0546	10	13	44	69.82	66.55	66.69	63.06	68.43
20091A0547	10	18	58	88.26	85.66	85.78	83.54	86.68
20091A0548	10	18.25	55	86.66	82.74	82.93	79.78	84.11
20091A0549	10	18.5	59	89.83	87.15	87.28	85.07	88.13
20091A0550	10	18.75	58	89.60	86.33	86.49	83.89	87.44
20091A0551	10	14.75	52	78.39	76.48	76.56	74.23	77.73
20091A0552	10	18.25	58	88.71	85.88	86.02	83.65	86.93
20091A0553	10	8.5	32	53.63	49.98	50.12	45.41	52.57
20091A0554	10	19	59	90.72	87.60	87.75	85.30	88.63
20091A0555	10	17.5	59	88.05	86.26	86.34	84.59	87.11
20091A0556	10	19	61	92.09	89.69	89.81	87.89	90.52
20091A0557	10	18.5	56	87.79	84.01	84.20	81.19	85.30
20091A0558	10	16.75	56	84.67	82.45	82.55	80.36	83.53
20091A0559	10	6.5	35	52.11	51.34	51.33	48.34	53.37
20091A0560	10	18.75	63	93.00	91.56	91.63	90.36	92.14
20091A0561	10	15.75	25	61.77	49.12	49.74	39.80	53.34
20091A0562	10	9.25	37	58.37	55.88	55.97	52.23	58.04
20091A0563	10	19	69	97.54	98.06	98.03	98.23	98.04
20091A0564	10	18.75	63	93.00	91.56	91.63	90.36	92.14
20091A0565	10	18.75	66	95.05	94.70	94.71	94.23	94.97
20091A0566	10	17.75	52	83.73	79.16	79.38	75.66	80.78
20091A0567	10	12.25	25	55.54	46.00	46.45	38.14	49.79
20091A0568	8.5	11	45	64.44	64.48	64.45	63.40	65.27
20091A0569	8.5	17.5	65	89.64	91.20	91.12	92.35	90.68
20091A0570	8.5	19	67	93.67	94.63	94.59	95.65	94.09
20091A0571	8	7.75	25	44.19	40.20	40.37	36.00	42.46
20091A0572	8.5	18.5	66	92.10	93.14	93.09	94.12	92.64
20091A0573	9.5	18.75	64	92.85	92.16	92.19	91.65	92.39
20091A0574	8.5	9.75	25	48.58	42.43	42.72	36.95	45.18
20091A0575	8.5	10.25	44	62.42	62.76	62.72	61.75	63.57
20091A0576	8.5	16.25	62	85.37	86.95	86.86	87.88	86.59
20091A0577	7.5	17.75	41	72.06	65.42	65.76	61.44	66.97

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20091A0578	9	19	66	93.82	94.03	94.02	94.35	93.84
20091A0579	9.5	8.5	25	48.03	42.21	42.47	36.36	45.29
20091A0580	8.5	11.5	25	51.70	43.99	44.36	37.78	46.95
20091A0581	8.5	15.75	58	81.75	82.32	82.28	82.47	82.32
20091A0582	8.5	15.75	59	82.43	83.36	83.31	83.76	83.26
20091A0583	9	13	43	67.47	64.61	64.74	61.77	66.11
20091A0584	8.5	17.75	60	86.68	86.19	86.22	86.00	86.23
20091A0585	8.5	7.75	13	36.85	28.09	28.50	20.49	31.86
20091A0586	8.5	16	57	81.52	81.49	81.49	81.29	81.63
20091A0587	8.5	15	38	66.79	60.72	61.02	56.25	62.74
20091A0588	8.5	13.5	43	67.53	64.61	64.75	62.01	65.92
20091A0589	8.5	18.25	63	89.61	89.78	89.77	90.12	89.56
20091A0590	8.5	11	44	63.76	63.43	63.42	62.11	64.33
20091A0591	8.5	10	40	59.25	58.35	58.37	56.46	59.55
20091A0592	8.5	14.25	53	75.67	75.75	75.73	75.29	76.09
20091A0593	8.5	18.5	56	85.29	82.68	82.81	81.19	83.23
20091A0595	8.5	18.75	65	91.86	92.32	92.30	92.94	91.95
20091A0596	8.5	8.25	42	57.50	58.89	58.78	58.22	59.65
20091A0597	9	18.5	65	92.25	92.54	92.53	92.82	92.39
20091A0598	8.5	12.75	33	59.38	53.48	53.76	48.72	55.75
20091A0599	9	18.75	59	88.61	86.48	86.59	85.19	87.00
20091A05A0	8	8	25	44.63	40.43	40.61	36.12	42.71
20091A05A1	8.5	11.75	43	64.41	63.05	63.10	61.17	64.15
20091A05A2	9.5	19	58	89.21	86.11	86.26	84.01	87.00
20091A05A3	9.5	6.25	9	33.12	23.46	23.90	14.60	27.95
20091A05A4	9	19	59	89.06	86.71	86.83	85.30	87.25
20091A05A5	9.5	17.75	60	88.35	87.08	87.14	86.00	87.61
20091A05A6	9	18.75	51	83.16	78.11	78.37	74.84	79.47
20091A05A7	8.5	11.75	37	60.32	56.77	56.93	53.42	58.50
20091A05A8	8.5	5	6	27.18	18.32	18.72	10.13	22.48
20091A05A9	9.5	18.5	63	91.72	90.89	90.93	90.24	91.20
20091A05B0	9.5	15	45	73.23	68.94	69.14	65.30	70.71
20091A05B1	9	19	59	89.06	86.71	86.83	85.30	87.25
20091A05B2	9	14.25	61	81.96	84.56	84.42	85.63	84.31
20091A05B3	9.5	19	64	93.30	92.38	92.43	91.77	92.65
20091A05B4	8.5	12.25	35	59.85	55.13	55.35	51.07	57.12
20091A05B5	8.5	13	39	63.91	59.98	60.16	56.60	61.65
20091A05B6	8.5	8.5	40	56.58	57.02	56.96	55.75	58.03
20091A05B7	8.5	16.5	58	83.09	82.98	82.99	82.82	83.08
20091A05B8	8.5	7.5	25	44.58	40.43	40.60	35.88	42.89
20091A05B9	8.5	1.5	13	25.72	22.52	22.62	17.52	25.51

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20091A05C0	7.5	13.5	42	65.17	62.68	62.80	60.71	63.60
20091A05C1	8.5	14.5	53	76.12	75.97	75.97	75.41	76.35
20091A05C2	10	18.75	62	92.32	90.51	90.60	89.06	91.20
20091A05C3	8.5	17.75	59	85.99	85.15	85.19	84.71	85.29
20091A05C4	8.5	10.5	38	58.78	56.71	56.79	54.12	58.17
20091A05C5	8.5	16.5	52	79.00	76.71	76.82	75.07	77.43
20091A05C6	9	18.5	57	86.80	84.17	84.30	82.48	84.86
20091A05C7	9	18	55	84.55	81.63	81.78	79.66	82.47
20091A05C8	8.5	7.5	11	35.04	25.78	26.21	17.78	29.72
20091A05C9	8.5	7.25	8	32.55	22.42	22.89	13.79	26.64
20091A05D0	8.5	17.25	61	86.47	86.79	86.78	87.06	86.67
20091A05D1	8.5	18.75	61	89.14	88.13	88.19	87.77	88.19
20091A05D2	8.5	17.75	57	84.63	83.05	83.14	82.12	83.41
20091A05D3	9.5	15	43	71.87	66.84	67.08	62.72	68.83
20091A05D4	9	8	25	46.30	41.32	41.53	36.12	44.09
20091A05D5	10	17	53	83.08	79.54	79.70	76.60	80.96
20091A05D6	9.5	7.5	25	46.25	41.32	41.53	35.88	44.28
20091A05D7	9.5	5.75	13	34.95	27.20	27.54	19.54	31.21
20091A05D8	10	17.25	50	81.48	76.62	76.85	72.84	78.39
20091A05D9	10	17	61	88.53	87.91	87.93	86.94	88.49
20091A05E0	10	18	63	91.67	90.89	90.92	90.00	91.38
20091A05E1	10	13.75	54	77.97	77.68	77.68	76.35	78.60
20091A05E2	10	18.75	59	90.28	87.37	87.52	85.19	88.38
20091A05E3	10	18.5	59	89.83	87.15	87.28	85.07	88.13
20091A05E4	10	16.75	52	81.95	78.27	78.44	75.19	79.76
20091A05E5	10	18.25	56	87.34	83.79	83.96	81.07	85.05
20091A05E6	10	17.25	0	47.41	24.30	25.45	8.20	31.33
20091A05E7	10	18	44	78.72	71.01	71.39	65.44	73.50
20091A05E8	9	11	40	61.86	59.69	59.77	56.94	61.25
20091A05E9	9.5	8	37	55.31	54.32	54.33	51.64	56.08
20091A05F0	10	16	57	84.02	82.83	82.87	81.29	83.71
20091A05F1	9	9.75	53	68.50	72.18	71.96	73.15	72.22
20091A05F2	10	18	47	80.77	74.15	74.48	69.31	76.33
20091A05F3	10	15.5	45	74.95	69.83	70.07	65.54	71.91
20091A05F4	10	17	54	83.76	80.58	80.73	77.89	81.90
20091A05F5	10	13.5	52	76.16	75.37	75.38	73.64	76.47
20091A05F6	10	17.75	60	89.18	87.53	87.60	86.00	88.31
20091A05F7	10	15.5	58	83.81	83.43	83.43	82.35	84.14
20091A05F8	10	16.25	57	84.47	83.05	83.11	81.41	83.96
20091A05F9	10	15.5	58	83.81	83.43	83.43	82.35	84.14
20091A05G0	10	15	55	80.88	79.85	79.88	78.23	80.81

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20091A05G1	9	12.75	41	65.66	62.30	62.45	59.06	63.97
20091A05G2	10	18.25	56	87.34	83.79	83.96	81.07	85.05
20091A05G3	10	15.25	59	84.05	84.25	84.23	83.52	84.83
20091A05G4	10	6	26	45.09	41.47	41.60	36.46	44.39
20091A05G5	10	17.25	61	88.97	88.13	88.16	87.06	88.74
20091A05G6	10	17.75	55	85.77	82.30	82.46	79.54	83.60
20091A05G7	10	18.25	60	90.07	87.98	88.07	86.24	88.81
20091A05G8	9	15	52	77.16	75.81	75.87	74.35	76.60
20091A05G9	10	18.5	62	91.88	90.29	90.37	88.94	90.95
20091A05H0	10	18	54	85.54	81.47	81.67	78.36	82.91
20091A05H1	10	19	66	95.49	94.92	94.95	94.35	95.22
20091A05H2	10	18.5	59	89.83	87.15	87.28	85.07	88.13
20091A05H3	8.5	14.5	58	79.53	81.20	81.11	81.87	81.05
20091A05H4	10	18.75	64	93.69	92.61	92.66	91.65	93.09
20091A05H5	9.5	6.5	34	50.60	49.85	49.84	47.04	51.73
20091A05H6	9.5	6.5	25	44.47	40.43	40.58	35.41	43.26
20091A05H7	9.5	5.75	25	43.13	39.76	39.88	35.05	42.50
20091A05H8	10	16.75	61	88.08	87.68	87.69	86.82	88.23
20091A05H9	10	18.25	56	87.34	83.79	83.96	81.07	85.05
20091A05J0	10	18.75	62	92.32	90.51	90.60	89.06	91.20
20091A05J1	10	17.75	58	87.82	85.44	85.55	83.42	86.42
20091A05J2	10	17.75	61	89.86	88.58	88.63	87.30	89.25
20091A05J3	10	17.75	62	90.54	89.62	89.66	88.59	90.19
20091A05J4	10	18	61	90.31	88.80	88.87	87.41	89.50
20091A05J5	10	17.75	63	91.22	90.67	90.69	89.88	91.13
20091A05J6	10	16.25	45	76.29	70.50	70.77	65.90	72.67
20091A05J7	10	17	57	85.80	83.72	83.82	81.77	84.72
20091A05J8	9	12.75	48	70.43	69.62	69.64	68.11	70.56
20091A05J9	9	9.25	28	50.57	45.57	45.79	40.59	48.19
20091A05K0	10	15.25	52	79.28	76.93	77.03	74.47	78.24
20091A05K1	10	13.75	46	72.52	69.31	69.45	66.00	71.07
20091A05K2	10	18.25	61	90.75	89.02	89.10	87.53	89.75

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PO Attainment: Mention first year courses

Table 8.5.1.1: I Year – I Semester - 2020 – 2021 Batch

Course	Course Title	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
A0001201	LA&AC	2.78	2.79	2.78	2.78	2.79	--	--	---	--	--	--	--

Actions taken based on the results of evaluation of relevant POs and PSOs

(The attainment levels by direct (student performance) are to be presented through Program level Course-PO matrix as indicated)

PO Attainment Levels and Actions for improvement – CAY only – Mention for relevant POs

POs	Target Level	Attainment Level	Observations
PO1:Engineering knowledge Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems			
PO1	2.5	2.78	Overall attainment is Extremely good when compared with the target level. This indicates that the majority students have good knowledge in Engineering
Action taken:			
PO2: Problem analysis Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences			
PO2	2.5	2.79	Overall attainment is Extremely good when compared with the target level. This indicates that the majority students have good problem solving skills.
Action taken:			
PO3:Design/ development of solutions Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations			
PO3	2.5	2.78	Overall attainment is Extremely good when compared with the target level. This indicates that the majority students are able to synthesize the problems
Action taken:			
PO4: Conduct investigations of complex problems: Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions			
PO4	2.5	2.78	Overall attainment is Extremely good when compared with the target level. This indicates that the majority students are able to understand and apply their knowledge to solve the problems with complexity.
Action taken:			
PO5: Modern tool usage: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations			
PO5	2.5	2.79	Overall attainment is Extremely good when compared with the target level. This indicates that the majority students are able to understand and apply their knowledge in developing the software related tools.
Action taken:			

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POs	Target Level	Attainment Level	Observations
PO6: The engineer and society: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice			
PO6	--	---	---
Action taken:			
PO7: Environment and sustainability: Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development			
PO7	---	---	---
Action taken:			
PO8: Ethics: Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.			
PO8	---	---	---
Action taken:			
PO9: Individual and teamwork: Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings			
PO9	---	---	---
Action taken:			
PO10: Communication: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions			
PO10	---	---	---
Action taken:			
PO11: Project management and finance: Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments			
PO11	---	---	---
Action taken:			
PO12: Life-long learning: Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change			
PO12	---	---	---
Action taken:			

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Outcome: It is observed that the students performance is good based on the attainment level.

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Sample T-L resources

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UNIT-I
Matrices

The term 'matrix' was introduced by great mathematician 'Cayley' in 1860. By matrix means an arrangement (or) rectangular array of numbers. Matrices find the applications in a solution of system of linear equations, probability, mathematical economics, quantum mechanics, electrical networks, curve fitting, transportation problems etc.

Matrices are easily applicable in different ways. In this chapter, a brief revision of matrices, types of matrices and their properties are presented.

Definition: The systematical arrangement of elements in square (or) rectangular array in which every row has the same number of elements and every column has the same number of elements is called a matrix.

It is denoted by [ ]

A set of mxn numbers arranged in the form of rectangular array having 'm' rows and 'n' columns is called a matrix of order mxn.

A mxn matrix is visually represented as

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & & & & \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix} = [a_{ij}]_{mn}$$

where a_{ij} is the element which lies at the intersection of i^{th} row and j^{th} column.

Types of matrices:

1. **Rectangular matrix:** A matrix in which the no. of rows are not equal to the no. of columns.

Ex: $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times 3}$

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2. Square matrix: A matrix in which the no. of rows are no. of columns

Ex: $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times3}$ $\begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times2}$

3. Row matrix:- A matrix having only one row and any no of columns

Ex: $[1 \ 2 \ 3]_{1\times3}$

4. Column matrix: A matrix having only one column and 'n' no. of rows.

Ex: $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{3\times1}$

5. Diagonal matrix: A square matrix in which all the elements except the diagonals are zeroes

Ex: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}_{3\times3}$

6. Scalar matrix: It is a diagonal matrix in which all the diagonal elements are equal

Ex: $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}_{3\times3}$

7. Unit matrix: It is a diagonal matrix in which all the diagonal elements equal to "one". It is denoted by I_n

Ex: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times2}$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times3}$

8. Triangular matrix: A square matrix in which every element either above (or) below the principle diagonal are zero.

(i) Upper Triangular matrix: It is a square matrix in which every element below the principle diagonal are zero's

Ex: $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$

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(iii) Lower triangular matrix: It is a square matrix in which every element above the principle diagonal are zero's 87

Ex: $\begin{bmatrix} 1 & 0 & 0 \\ 4 & 5 & 0 \\ 6 & 7 & 1 \end{bmatrix}$

9. Transpose of a matrix: A matrix 'A' is obtained by interchanging their rows and columns is called a transpose of a matrix. It is denoted by A^T.

Ex: If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times3}$ then $A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}_{3\times2}$

10. Symmetric matrix: A square matrix is said to be a symmetric if $A^T = A$

Ex: Let $A = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$ $A^T = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$

Here $A^T = A$

$\therefore$ A is Symmetric matrix

11. Skew symmetric matrix: A square matrix is said to be a skew symmetric matrix if $A^T = -A$

Ex: $A = \begin{bmatrix} 0 & h & g \\ -h & 0 & f \\ -g & -f & 0 \end{bmatrix}$ $A^T = \begin{bmatrix} 0 & -h & -g \\ h & 0 & -f \\ g & f & 0 \end{bmatrix}$

Here $A^T = -A$

$\therefore$ A is skew symmetric matrix

12. Conjugate of a matrix: A matrix is obtained of a given matrix 'A' by replacing their corresponding conjugate complex numbers is called conjugate of a given matrix A.

It is denoted by $\bar{A}$.

Ex:

$A = \begin{bmatrix} 1 & 1+4i \\ -1+2i & 4 \end{bmatrix}$ then $\bar{A} = \begin{bmatrix} 1 & 1-4i \\ -1-2i & 4 \end{bmatrix}$

13. Transpose Conjugate matrix: The matrix obtained by taking the transpose of the conjugate of a matrix 'A' is called Transpose Conjugate matrix.

It is denoted by $(\bar{A})^T$ (or) A^θ

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Ex: $A = \begin{bmatrix} 1 & 1+4i \\ -1+3i & 4 \end{bmatrix}$; $\bar{A} = \begin{bmatrix} 1 & 1-4i \\ -1-3i & 4 \end{bmatrix}$

$(\bar{A})^T = \begin{bmatrix} 1 & -1-3i \\ 1-4i & 4 \end{bmatrix}$

Hermition matrix: A matrix is said to be Hermitian if A^θ

Ex: $A = \begin{bmatrix} a & a-ib \\ a+ib & d \end{bmatrix}$; $\bar{A} = \begin{bmatrix} a & a+ib \\ a-ib & d \end{bmatrix}$

$(\bar{A})^T = \begin{bmatrix} a & a-ib \\ a+ib & d \end{bmatrix}$

Here $(\bar{A})^T = A$

$\therefore$ It is a Hermition matrix

Skew Hermition matrix: A matrix is said to be skew Herm

if $A^\theta = -A$

Ex: $A = \begin{bmatrix} 0 & -2-i \\ 2-i & 0 \end{bmatrix}$; $\bar{A} = \begin{bmatrix} 0 & -2+i \\ 2+i & 0 \end{bmatrix}$

$(\bar{A})^T = \begin{bmatrix} 0 & 2+i \\ -2+i & 0 \end{bmatrix}$

Here $(\bar{A})^T = -A$

$\therefore$ It is a skew Hermition matrix

Orthogonal matrix: Let 'A' be the matrix said to be Orth

matrix if $AA^T = A^T A = I$

Idempotent matrix: Let 'A' be the given matrix is said to

idempotent if $A^2 = A$

Involuntary matrix: Let 'A' be the given matrix is said t

Involuntary if $A^2 = I$, where I is identity matrix

Nilpotent matrix: Let 'A' be the given matrix is said to

nilpotent if $A^n = 0$ where n = order of a matrix

Suppose if $A^2 = 0$ then matrix is nilpotent matrix of ord

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Eigen Values and Eigen Vectors

Let $A = [a_{ij}]_{m \times n}$ be a Square matrix, then

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$ and 'λ' is a scalar.

$$|A-\lambda I| = \begin{vmatrix} a_{11}-\lambda & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22}-\lambda & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn}-\lambda \end{vmatrix}$$ then the $[A-\lambda I]$ is

called Characteristic matrix of 'A'.

$|A-\lambda I| = 0$ is called characteristic equation of a matrix 'A'.

1. The roots of characteristic eqn are called characteristic roots (or) Eigen values.
2. The set of Eigen values is called spectrum of the matrix A.
3. The Corresponding Characteristic matrix for eigen values are called Eigen vectors of A.

Aswini Malini

Characteristic Vector:-

If 'λ' is the Eigen value of $m \times n$ matrix 'A' then a non zero vector 'X' which satisfies the equation $[A-\lambda I]x = 0$ i.e. $Ax = \lambda x$ is called characteristic vector of matrix 'A' corresponding to the Eigen values.

1. Find the characteristic (Eigen) value of matrix

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

Given, the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

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The characteristic eq'n of matrix A is

$$|A-\lambda I|=0$$

$$\begin{vmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{vmatrix}=0$$

$$(2-\lambda)\left[(3-\lambda)(2-\lambda)-2\right]-2\left((2-\lambda)-1\right)+1\left(2-(3-\lambda)\right)=0$$

$$(2-\lambda)\left[6-5\lambda+\lambda^2-2\right]-2(1-\lambda)+(-1+\lambda)=0$$

$$(2-\lambda)\left[\lambda^2-5\lambda+4\right]-2(1-\lambda)+(\lambda-1)=0$$

$$(2-\lambda)(\lambda-1)(\lambda-4)+2(\lambda-1)+(\lambda-1)=0 \qquad \lambda^2-4\lambda-\lambda+4, \quad \lambda(\lambda-4)-1(\lambda-4)$$

$$(\lambda-1)\left[(2-\lambda)(\lambda-4)+2+1\right]=0$$

$$\lambda-1\left[2\lambda+4\lambda-8-\lambda^2+3\right]=0$$

$$(\lambda-1)\left[-\lambda^2+6\lambda-5\right]=0$$

$$(\lambda-1)\left[\lambda^2-6\lambda+5\right]=0$$

$$\lambda-1=0 \qquad \lambda^2-6\lambda+5=0$$

$$\boxed{\lambda_1=1} \qquad \lambda^2-5\lambda-\lambda+5=0$$

$$\lambda(\lambda-5)-1(\lambda-5)=0$$

$$(\lambda-1)(\lambda-5)=0$$

$$\boxed{\lambda=1,5}$$

$\therefore$ Eigen values $=1,1,5$

$\therefore$ spectrum of $A=\{1,1,5\}$

2. Find the Eigen values and eigen vectors of matrix

$$A=\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

The given matrix $A=\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

The characteristic eq'n of matrix A is

$$|A-\lambda I|=0$$

$$\begin{vmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{vmatrix}=0$$

$(-2-\lambda)[-\lambda(1-\lambda)-12]-2[-2\lambda-6]-3[-4+1-\lambda]=0$

$(-2-\lambda)[-\lambda+\lambda^2-12]-2[-2\lambda-6]-3[-3-\lambda]=0$ 9/

$-(2+\lambda)[\lambda^2-\lambda-12]+4\lambda+12+9+3\lambda=0$

$-2\lambda^2+2\lambda+24-\lambda^3+\lambda^2+12\lambda+7\lambda+21=0$

$-\lambda^3-\lambda^2+21\lambda+45=0$

$\lambda^3+\lambda^2-21\lambda-45=0$ $\quad -27+9+63-45$

$$\begin{array}{r|rrrr} -3 & 1 & 1 & -21 & -45 \\ & 0 & -3 & 6 & 45 \\ \hline & 1 & -2 & -15 & 0 \end{array}$$

$\lambda^2-2\lambda-15=0$

$\lambda^2-5\lambda+3\lambda-15=0$

$\lambda(\lambda-5)+3(\lambda-5)=0$

$\lambda=5,\ \lambda=-3$

$\therefore$ Eigen Values $=-3,-3,+5$

$\therefore$ Spectrum of $A=\{-3,-3,5\}$

<u>To find the Eigen vectors</u>: Let 'x' be the vector for corresponding Eigen Value λ, such that $[A-\lambda I]x=0$

Case(i): if $\lambda=-3$

$$[A-\lambda I]=\begin{bmatrix}-2-\lambda & 2 & -3\\ 2 & 1-\lambda & -6\\ -1 & -2 & -\lambda\end{bmatrix}$$

$$[A-(-3)I]=\begin{bmatrix}1 & 2 & -3\\ 2 & 4 & -6\\ -1 & -2 & 3\end{bmatrix}$$

$[A-\lambda I]x=0$

$$\begin{bmatrix}1 & 2 & -3\\ 2 & 4 & -6\\ -1 & -2 & 3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$R_2\to R_2-2R_1$
$R_3\to R_3+R_1$
$$\begin{bmatrix}1 & 2 & -3\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

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The reduced equations are

$x_1+2x_2-3x_3=0$

No. of orbitary Constants $=n-r=3-1=2$

let $x_2=K_1$; $x_3=K_2$

$x_1+2K_1-3K_2=0$

$x_1=3K_2-2K_1$

$$\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}3K_2-2K_1\\K_1\\K_2\end{bmatrix}$$

$$=K_1\begin{bmatrix}-2\\1\\0\end{bmatrix}+K_2\begin{bmatrix}3\\0\\1\end{bmatrix}$$

Here ~~[-2,1,0]~~ $\begin{bmatrix}-2\\1\\0\end{bmatrix}$ and $\begin{bmatrix}3\\0\\1\end{bmatrix}$ are the Eigen vectors Corresponding to the Eigen value -3 and we can say that linear Combination of $\begin{bmatrix}-2\\1\\0\end{bmatrix}$ and $\begin{bmatrix}3\\0\\1\end{bmatrix}$ is also an Eigen vectors

Case (ii) : if $\lambda=5$

$[A-\lambda I]X=0$

$$\begin{bmatrix}-7&2&-3\\2&-4&-6\\-1&-2&-5\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$R_2\to 7R_2+2R_1$
$R_3\to 7R_3-R_1$

$$\begin{bmatrix}-7&2&-3\\0&-24&-48\\0&-16&-32\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$R_2\to \frac{R_2}{-24}$
$R_3\to \frac{R_3}{-16}$

$$\begin{bmatrix}-7&2&-3\\0&1&2\\0&1&2\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$R_3\to R_3-R_2$

$$\begin{bmatrix}-7&2&-3\\0&1&2\\0&0&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

Then the reduced equations are

$-7x_1+2x_2-3x_3=0 \to (1)$

$x_2+x_3=0 \to (2)$

BETA AND GAMMA FUNCTIONS

Beta function

The first eulerian integral $B(x,y)=\int_0^1 t^{x-1}(1-t)^{y-1}\,dt$ where m>0, n>0 is called a Beta function and is denoted by B(m,n).

The quantities m and n are positive but not necessarily integers.

Example:-

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Properties of Beta Function

$$B(x,y) = B(y,x).$$

$$B(x,y) = \int_0^\infty \frac{t^{x-1}}{(1+t)^{x+y}}\,dt, \qquad \mathrm{Re}(x) > 0,\ \mathrm{Re}(y) > 0$$

$$B(x,y) = B(x,y+1) + B(x+1,y)$$

$$xB(x,y+1) = y\,B(x+1,y)$$

$$B(x,y) = 2\int_0^{\pi/2} (\sin\theta)^{2x-1}(\cos\theta)^{2y-1}\,d\theta, \qquad \mathrm{Re}(x) > 0,\ \mathrm{Re}(y) > 0$$

$$B(x,y) = \frac{\Gamma(x)\,\Gamma(y)}{\Gamma(x+y)}$$

$$B(x,y)\cdot B(x+y,1-y) = \frac{\pi}{x\sin(\pi y)},$$

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Gamma function

The Eulerian integral $\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}dt$,n>0 is called gamma function and is denoted by $\Gamma(x)$

Example:- $\Gamma(1) = \int_0^\infty t^{1-1}e^{-t}dt$

$$= \lim_{n\to\infty}\int_0^n e^{-t}\,dt$$

$$= \lim_{n\to\infty} -e^{-t}\Big|_0^n$$

$$= \lim_{n\to\infty} \frac{-1}{e^n} - \frac{-1}{e^0} = \lim_{n\to\infty} 1 - \frac{1}{e^n} = 1$$

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Quadratic Form

A homogeneous equation of the second degree in any no. of variables is called a quadratic form.

(or)

A homogeneous polynomial of degree '2' in 'n' variables is called a quadratic form

Ex 1: $ax^2+2hxy+by^2$ is a quadratic form in two variables x and y.

2. $ax^2+by^2+cz^2+2hxy+2gxz+2fyz$ is a quadratic form in three variables x, y and z

* In general, the quadratic form is $Q=\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}x_i x_j$
* An expression is of the form $Q=X^TAX$

$$= \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}x_i x_j$$

Where a_{ij} are constants is called a quadratic form in 'n' variables $x_1, x_2, \ldots x_n$. If the constants a_{ij}'s are real numbers then it is called a real quadratic form.

$$Q = X^TAX = \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}x_i x_j \quad [i \neq j]$$

$$Q = [x_1\ x_2\ x_3 \ldots x_n]\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{n1} & a_{n2} & \cdots & a_{nn}\end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n\end{bmatrix}$$ is known as the matrix form of a quadratic form.

Consider the quadratic form $Q=a_{11}x_1^2+2a_{12}x_1x_2+a_{22}x_2^2$

The above quadratic form in the matrix form can be represented as

$$Q = a_{11}x_1^2+(a_{12}+a_{21})x_1x_2+a_{22}x_2^2$$

$$\begin{array}{c|cc} & x_1 & x_2 \\ \hline x_1 & a_{11} & a_{12} \\ x_2 & a_{21} & a_{22}\end{array}$$

$$= [x_1\ x_2]\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22}\end{bmatrix}\begin{bmatrix} x_1 \\ x_2\end{bmatrix}$$

$$Q = X^TAX$$

where $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22}\end{bmatrix}$

A homogeneous

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2. $Q = a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + 2a_{12}x_1x_2 + 2a_{13}x_1x_3 + 2a_{23}x_2x_3$

$Q = a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + (a_{12}+a_{21})x_1x_2 + (a_{13}+a_{31})x_1x_3 + (a_{23}+a_{32})x_2x_3$

$$= [x_1 \;\; x_2 \;\; x_3]\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$Q = X^T A X$

Here $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

3. If $A = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$ & $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ then write the quadratic form.

The general quadratic form is given by

$$Q = X^T A X$$

$$Q = [x_1 \;\; x_2 \;\; x_3]\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$Q = \begin{bmatrix} ax_1 + hx_2 + gx_3 \\ hx_1 + bx_2 + fx_3 \\ gx_1 + fx_2 + cx_3 \end{bmatrix}[x_1 \;\; x_2 \;\; x_3]$$

$$Q = (ax_1 + hx_2 + gx_3)x_1 + (hx_1 + bx_2 + fx_3)x_2 + (gx_1 + fx_2 + cx_3)x_3$$

$$Q = ax_1^2 + hx_1x_2 + gx_1x_3 + hx_1x_2 + bx_2^2 + fx_3x_2 + gx_1x_3 + fx_2x_3 + cx_3^2$$

$$Q = ax_1^2 + bx_2^2 + cx_3^2 + 2hx_1x_2 + 2fx_2x_3 + 2gx_1x_3$$

4. Find the symmetric matrix corresponding to the quadratic form $x_1^2 + 6x_1x_2 + 5x_2^2$

Given $Q = x_1^2 + 6x_1x_2 + 5x_2^2$

$Q = x_1^2 + 3x_1x_2 + 3x_2x_1 + 5x_2^2$

	x_1	x_2
x_1	1	3
x_2	3	5

$$Q = [x_1 \;\; x_2]\begin{bmatrix} 1 & 3 \\ 3 & 5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$Q = x^T A x$$

Here, $A = \begin{bmatrix} 1 & 3 \\ 3 & 5 \end{bmatrix}$ is a symmetric matrix.

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(ii) $x^2+2y^2+3z^2+4xy+5yz+6zx$

$Q = x^2+2y^2+3z^2+4xy+5yz+6zx$

$Q = x^2+2y^2+3z^2+2xy+2yx+\frac{5}{2}yz+\frac{5}{2}zy+3zx+3xz$

	x	y	z
x	1	2	3
y	2	2	$\frac{5}{2}$
z	3	$\frac{5}{2}$	3

$A = \begin{bmatrix}1 & 2 & 3\\ 2 & 2 & 5/2\\ 3 & 5/2 & 3\end{bmatrix}$ is a symmetric matrix.

(iii) $x_1^2+2x_2^2+4x_2x_3+x_3x_4$

Given $Q = x_1^2+2x_2^2+4x_2x_3+x_3x_4$

$Q = x_1^2+2x_2^2+2x_2x_3+2x_3x_2+\frac{1}{2}x_3x_4+\frac{1}{2}x_4x_3$

	x_1	x_2	x_3	x_4
x_1	1	0	0	0
x_2	0	2	2	0
x_3	0	2	0	1/2
x_4	0	0	1/2	0

$A = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 2 & 2 & 0\\ 0 & 2 & 0 & 1/2\\ 0 & 0 & 1/2 & 0\end{bmatrix}$ is a symmetric matrix.

5. Find the quadratic form for the matrix $\begin{bmatrix}1 & 2 & 3\\ 2 & 1 & 3\\ 3 & 3 & 1\end{bmatrix}$

Given, the matrix A is a symmetric matrix

Let $X = [x\ y\ z]$

We know that $Q = X^TAX$

$Q = \begin{bmatrix}x\\ y\\ z\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 2 & 1 & 3\\ 3 & 3 & 1\end{bmatrix}[x\ \ y\ \ z]$

$Q = \begin{bmatrix}x+2y+3z\\ 2x+y+3z\\ 3x+3y+z\end{bmatrix}[x\ \ y\ \ z]$

$Q = x(x+2y+3z)+y(2x+y+3z)+z(3x+3y+z)$

$Q = x^2+2xy+3xz+2yx+y^2+3yz+3zx+3zy+z^2$

$Q = x^2+y^2+z^2+4xy+6xz+6yz$

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6. Write the quadratic form for given matrices.

(i) $A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 0 & 3 \\ 5 & 3 & 4 \end{bmatrix}$ (ii) $A = \begin{bmatrix} 0 & 5 & -1 \\ 5 & 1 & 6 \\ -1 & 6 & 2 \end{bmatrix}$ (iii) $A = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \end{bmatrix}$

(iv) $A = \begin{bmatrix} 2 & -3 & 5 \\ -3 & 2 & -2 \\ 5 & -2 & 2 \end{bmatrix}$

Rank of a quadratic form:

Let X^TAX be a quadratic form over a real number R the rank 'R' of a matrix 'A' is called the rank of a quadratic form (or) The total no of square terms (both +ve & -ve)

Linear transformation of a quadratic form:

Let $Q = X^TAX$ is transformed to another quadratic form Y^TBY is said to be a linear transformation it can be written as $X = PY$.

Let X^TAX be the given quadratic form then

$$X^TAX = (PY)^TA(PY)$$

$$X^TAX = Y^T(P^TAP)Y$$

$$X^TAX = Y^TBY \quad [\because B = P^TAP]$$

B is a diagonal form

Here Y^TBY is also a quadratic form but B is the diagonal matrix.

Hence, one quadratic form is transformed into another quadratic form such that $X = PY$.

Canonical form (or) Normal form of a quadratic form:

A real quadratic form in which the product terms are missing and in which only it contains the terms of squares of variables is called a canonical or normal form

Ex: $\lambda_1 y_1^2 + \lambda_2 y_2^2 + \lambda_3 y_3^2 + \dots + \lambda_n y_n^2$

(or)

Let X^TAX be the quadratic form of (n) variables then there exist a real non-singular linear transformation $X = PY$ which transforms X^TAX to the another quadratic form of a type Y^TDY. Here $Y^TDY = \lambda_1 y_1^2 + \lambda_2 y_2^2 + \lambda_3 y_3^2 + \dots$ where D is the diagonal matrix.

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Unit - IV

a) Mean Value Theorems

Continuous Functions: A continuous function is a real valued function whose graph does not have any breaks. In other words, $f(x)$ is said to be continuous function if it exists at every point in the given intervals.

Differential Function: $f(x)$ is said to be differentiable function if $f'(x)$ exists at every point of the given interval.

Rolle's Theorem:- Let $f(x)$ be a function such that

(i) it is continuous in closed interval $[a,b]$.

(ii) it is differential in open interval (a,b) and

(iii) $f(a) = f(b)$.

Then there exists atleast one point c in open (a,b) such that $f'(c) = 0$.

* Verify Rolle's theorem for $f(x) = (x+2)^3(x-3)^4$ in $[-2,3]$.

sol:- given $f(x) = (x+2)^3(x-3)^4$ in $[-2,3]$

(i) $f(x)$ exists for all values of $x \in [-2,3]$

$\therefore f(x)$ is continuous in $[-2,3]$.

(ii) $f'(x) = 3(x+2)^2(x-3)^4 + 4(x+2)^3(x-3)^3$.

$= (x+2)^2(x-3)^3[3x-9+4x+8]$

$= (x+2)^2(x-3)^3(7x-1)$.

$\therefore f'(x)$ exists for all values of $x \in (-2,3)$.

$\therefore f(x)$ is differentiable in $(-2,3)$.

(iii) $f(-2) = (-2+2)^3(-2-3)^3 = 0$

$f(3) = (3+2)^3(3-3)^4 = 0$.

$f(-2) = f(3)$

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All the conditions of Rolle's Theorem are satisfied. Therefore there exist atleast one point $c \in (a, b)$ such that $f'(c)=0$

i.e $(c+2)^2(c-3)^3(7c-1)=0$.

$\Rightarrow c = -2, 3, 1/7$.

Clearly $c = \frac{1}{7} \in (-2, 3)$.

Hence Rolle's theorem is verified.

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* Verify Rolle's theorem for $f(x) = 2x^3+x^2-4x-2$ in $[-\sqrt{2}, \sqrt{2}]$.

<u>sol:</u>- given $f(x) = 2x^3+x^2-4x-2$ in $[-\sqrt{2}, \sqrt{2}]$.

here $f(x)$ is a polynomial and hence it is continuous and differentiable for all x. In particular

(i) $f(x)$ is continuous in $[-\sqrt{2}, \sqrt{2}]$

(ii) $f(x)$ is differentiable in $(-\sqrt{2}, \sqrt{2})$

and $f'(x) = 6x^2+2x-4$.

(iii) $f(-\sqrt{2}) = 2(-\sqrt{2})^3 + (-\sqrt{2})^2 + 4\sqrt{2} - 2$

$= -4\sqrt{2} + 2 + 4\sqrt{2} - 2 = 0$

$f(\sqrt{2}) = 2(\sqrt{2})^3 + (\sqrt{2})^2 - 4\sqrt{2} - 2$

$= 4\sqrt{2} + 2 - 4\sqrt{2} - 2 = 0$.

$f(-\sqrt{2}) = f(\sqrt{2})$.

$\therefore$ All the conditions of Rolle's theorem are satisfied.

$\therefore$ There exist atleast one point $c \in (-\sqrt{2}, \sqrt{2})$ such that $f'(c) = 0$.

i.e $6c^2+2c-4=0$.

$\Rightarrow 3c^2+c-2=0$

$\Rightarrow c = \frac{-(1) \pm \sqrt{1^2-4(3)(-2)}}{2(3)}$

$= \frac{-1 \pm \sqrt{1+24}}{6} = \frac{-1 \pm \sqrt{25}}{6} = \frac{-1 \pm 5}{6}$

$c = \frac{-1-5}{6}, \frac{-1+5}{6} = \frac{-6}{6}, \frac{4}{6} = -1, 2/3$

if $ax^2+bx+c=0$ then $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$.

Clearly $c=-1$, and $c = 2/3$ both are in the interval $(-\sqrt{2}, \sqrt{2})$. Hence Theorem verified.

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* Verify Rolle's Theorem for the function $f(x) = \frac{\sin x}{e^x}$ in $[0,\pi]$.

Sol:- Given that $f(x) = \frac{\sin x}{e^x}$ in $[0,\pi]$.

(i) $\sin x$ and e^x both are continuous functions in $[0,\pi]$.

$f(x)$ exist for all values of $x \in [0,\pi]$.

$\therefore f(x)$ is continuous in $[0,\pi]$.

(ii) $$f'(x) = \frac{e^x \cos x - \sin x \cdot e^x}{(e^x)^2}$$

$$= \frac{e^x[\cos x - \sin x]}{(e^x)^2} = \frac{\cos x - \sin x}{e^x}.$$

which exists for all values of $x \in (0,\pi)$.

$\therefore f(x)$ is differentiable in $(0,\pi)$.

(iii) $$f(0) = \frac{\sin(0)}{e^0} = \frac{0}{1} = 0.$$

$$f(\pi) = \frac{\sin\pi}{e^\pi} = \frac{0}{e^\pi} = 0.$$

$\therefore f(0) = f(\pi)$.

All the conditions of Rolle's theorem are verified.

$\therefore$ There exist atleast one point $c \in (0,\pi)$ such that $f'(c)=0$.

i.e $\frac{\cos c - \sin c}{e^c} = 0$, $\Rightarrow \cos c - \sin c = 0$

$\Rightarrow \cos c = \sin c$

$\Rightarrow \tan c = 1 \Rightarrow c = \pi/4$.

Clearly $c = \frac{\pi}{4} \in (0,\pi)$.

Hence Rolle's theorem is verified.

* Verify Rolle's theorem for the function
$f(x) = (x-a)^m(x-b)^n$ where m,n are positive integers in $[a,b]$.

Sol:- Given that $f(x) = (x-a)^m(x-b)^n$. in $[a,b]$.

(i) $f(x)$ exists for all values of $x \in [a,b]$

$\therefore f(x)$ is continuous in $[a,b]$.

(ii) $$f'(x) = m(x-a)^{m-1}(x-b)^n + n.(x-a)^m(x-b)^{n-1}$$

$$= (x-a)^{m-1}(x-b)^{n-1}[m(x-b) + n(x-a)]$$

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$$= (x-a)^{m-1}(x-b)^{n-1}[(m+n)x-(mb+na)].$$

which exists for all values of $x \in (a,b)$.

(iii) $f(a) = (a-a)^m(a-b)^n = 0$

$f(b) = (b-a)^m(b-b)^n = 0$.

$\therefore f(a) = f(b)$.

All the conditions of Rolle's theorem are satisfied.

$\therefore$ There exist atleast one point $c \in (a,b)$ such that

$$f'(c) = 0.$$

i.e $(c-a)^{m-1}(c-b)^{n-1}[(m+n)c-(mb+na)] = 0$.

$$\Rightarrow c = a,\ c = b,\ c = \frac{mb+na}{m+n}.$$

clearly $c = \dfrac{mb+na}{m+n} \in [a,b]$

Hence the Rolle's theorem is verified.

* Verify Rolle's theorem for the function $\log\left[\dfrac{x^2+ab}{x(a+b)}\right]$ in $[a,b]$ $a>0, b>0$.

sol:- ~~Given that~~ let $f(x) = \log\left[\dfrac{x^2+ab}{x(a+b)}\right]$

$$= \log(x^2+ab) - \log[x(a+b)]$$

$$= \log(x^2+ab) - \log x - \log(a+b).$$

(i) $f(x)$ exists for all values of $x \in [a,b]$

$\therefore f(x)$ is continuous ~~for all~~ in $[a,b]$; $a>0, b>0$.

(ii) $f'(x) = \dfrac{1}{(x^2+ab)}(2x) - \dfrac{1}{x} - 0 = \dfrac{2x}{x^2+ab} - \dfrac{1}{x} = \dfrac{x^2-ab}{x(x^2+ab)}$

which exists for all values of $x \in (a,b)$, $a>0, b>0$

$\therefore f(x)$ is differentiable in (a,b).

(iii) $f(a) = \log\left[\dfrac{a^2+ab}{a(a+b)}\right] = \log\left[\dfrac{a^2+ab}{a^2+ab}\right] = \log(1) = 0$.

$f(b) = \log\left[\dfrac{b^2+ab}{b(a+b)}\right] = \log\left[\dfrac{b^2+ab}{ab+b^2}\right] = \log\left[\dfrac{b^2+ab}{b^2+ab}\right] = \log(1) = 0$.

$f(a) = f(b)$.

$\therefore$ All the conditions of Rolle's theorem

Unit - V

Multiple Integrals

Double Integral: Let us consider a function of two variables $z = f(x,y)$. Then the double integral of the function $f(x,y)$ is denoted by

$$\iint_R f(x,y)\,dx\,dy$$

where R is the region of integration in the xy plane.

* Evaluate $\int_0^2 \int_0^x y\,dy\,dx$.

Sol:- $\int_0^2 \int_0^x y\,dy\,dx = \int_{x=0}^2 \left[\int_{y=0}^x y\,dy\right] dx$

$$= \int_{x=0}^2 \left[\frac{y^2}{2}\right]_{y=0}^x dx = \int_{x=0}^2 \left[\frac{x^2}{2} - \frac{0^2}{2}\right] dx$$

$$= \int_{x=0}^2 \frac{x^2}{2}\,dx = \frac{1}{2}\left[\frac{x^3}{3}\right]_0^2 = \frac{1}{6}\left(2^3 - 0^3\right) = \frac{1}{6}(8-0) = 4/3.$$

* Evaluate $\int_{y=0}^2 \int_{x=0}^3 xy\,dx\,dy$.

Sol:- $\int_{y=0}^2 \int_{x=0}^3 xy\,dx\,dy = \int_{y=0}^2 y\left[\int_{x=0}^3 x\,dx\right] dy$

$$= \int_{y=0}^2 y\left(\frac{x^2}{2}\right)_{x=0}^3 dy = \int_{y=0}^2 y\left(\frac{3^2}{2} - \frac{0^2}{2}\right) dy$$

$$= \int_{y=0}^2 y\left(\frac{9}{2} - 0\right) dy = \frac{9}{2}\int_{y=0}^2 y\,dy = \frac{9}{2}\left(\frac{y^2}{2}\right)_0^2$$

$$= \frac{9}{2}\left(\frac{2^2}{2} - 0\right) = \frac{9}{2}(2-0) = 9.$$

Alternate Method: Here x and y are independent.

$$\therefore \int_{y=0}^2 \int_{x=0}^3 xy\,dx\,dy = \left[\int_{y=0}^2 y\,dy\right]\left[\int_{x=0}^3 x\,dx\right]$$

$$= \left(\frac{y^2}{2}\right)_0^2 \left(\frac{x^2}{2}\right)_0^3 = \left(\frac{4}{2}\right)\left(\frac{9}{2}\right)$$

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* Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2+y^2)\,dx\,dy$

Sol:- $\int_0^1 \int_x^{\sqrt{x}} (x^2+y^2)\,dx\,dy = \int_{x=0}^{1} \int_{y=x}^{\sqrt{x}} (x^2+y^2)\,dy\,dx.$

$$= \int_{x=0}^{1} \left(x^2 y + \frac{y^3}{3}\right)_{y=x}^{\sqrt{x}} dx.$$

$$= \int_{x=0}^{1} \left(x^2 \cdot \sqrt{x} + \frac{(\sqrt{x})^3}{3} - x^2 \cdot x - \frac{x^3}{3}\right) dx.$$

$$= \int_{x=0}^{1} \left(x^{2+\frac{1}{2}} + \frac{x^{3/2}}{3} - x^3 - \frac{x^3}{3}\right) dx$$

$$= \int_{x=0}^{1} \left(x^{5/2} + \frac{x^{3/2}}{3} - \frac{4}{3}x^3\right) dx$$

$$= \left[\frac{x^{5/2+1}}{5/2+1} + \frac{x^{3/2+1}}{3(3/2+1)} - \frac{4}{3}\frac{x^4}{4}\right]_0^1$$

$$= \left[\frac{x^{7/2}}{7/2} + \frac{x^{5/2}}{3(5/2)} - \frac{1}{3}x^4\right]_0^1 = \left(\frac{2}{7}x^{7/2} + \frac{2}{15}x^{5/2} - \frac{x^4}{3}\right)_0^1$$

$$= \frac{2}{7} + \frac{2}{15} - \frac{1}{3} = \frac{30+14-35}{105} = \frac{9}{105} = \frac{3}{35}.$$

* Evaluate $\int_0^a \int_0^{\sqrt{a^2-y^2}} \sqrt{a^2-x^2-y^2}\,dx\,dy$

Sol:- $\int_0^a \int_0^{\sqrt{a^2-y^2}} \sqrt{a^2-x^2-y^2}\,dx\,dy = \int_{y=0}^{a} \int_{x=0}^{\sqrt{a^2-y^2}} \sqrt{(\sqrt{a^2-y^2})^2 - x^2}\,dx\,dy$

$$= \int_{y=0}^{a} \left[\frac{x}{2}\sqrt{a^2-y^2-x^2} + \frac{a^2-y^2}{2}\sin^{-1}\left(\frac{x}{\sqrt{a^2-y^2}}\right)\right]_{x=0}^{\sqrt{a^2-y^2}} dy$$

$$\left[\because \int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]$$

$$= \int_{y=0}^{a} \left[\frac{\sqrt{a^2-y^2}}{2}\sqrt{a^2-y^2-a^2+y^2} + \frac{(a^2-y^2)}{2}\sin^{-1}\left(\frac{\sqrt{a^2-y^2}}{\sqrt{a^2-y^2}}\right) - 0 - \frac{(a^2-y^2)}{2}\sin^{-1}(0)\right] dy$$

$$= \int_{y=0}^{a} \left[\frac{\sqrt{a^2-y^2}}{2}(0) + \left(\frac{a^2-y^2}{2}\right)\sin^{-1}(1) - \left(\frac{a^2-y^2}{2}\right)\sin^{-1}(0)\right] dy$$

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$$= \int_{y=0}^{a} \frac{(a^2-y^2)}{2}\left(\frac{\pi}{2}\right) dy = \frac{\pi}{4}\int_{y=0}^{a} (a^2-y^2)\, dy$$

$$= \frac{\pi}{4}\left(a^2y - \frac{y^3}{3}\right)_0^a = \frac{\pi}{4}\left(a^2\cdot a - \frac{a^3}{3}\right) = \frac{\pi}{4}\left(a^3-\frac{a^3}{3}\right) = \frac{\pi}{4}\,\frac{2a^3}{3}$$

$$= \frac{\pi}{6}a^3.$$

* Evaluate $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dy\,dx}{1+x^2+y^2}$.

Sol:- $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dy\,dx}{1+x^2+y^2} = \int_{x=0}^{1} \int_{y=0}^{\sqrt{1+x^2}} \frac{dy}{(1+x^2)+y^2}\, dx.$

$$= \int_{x=0}^{1} \int_{y=0}^{p} \frac{dy}{p^2+y^2}\, dx \text{ where } p = \sqrt{1+x^2}.$$

$$= \int_{x=0}^{1} \left[\frac{1}{p}\,\mathrm{Tan}^{-1}\left(\frac{y}{p}\right)\right]_{y=0}^{p} dx = \int_{x=0}^{1} \frac{1}{p}\left[\mathrm{Tan}^{-1}(1) - \mathrm{Tan}^{-1}(0)\right] dx$$

$$= \int_{x=0}^{1} \frac{\pi}{4}\left(\frac{dx}{\sqrt{1+x^2}}\right) \quad [\because p = \sqrt{1+x^2}]$$

$$= \frac{\pi}{4}\left[\log\left(x+\sqrt{x^2+1}\right)\right]_{x=0}^{1} = \frac{\pi}{4}\log(1+\sqrt{2})$$

* Evaluate $\int_0^2 \int_0^x e^{x+y}\, dy\, dx$.

Sol:- $\int_0^2 \int_0^x e^{x+y}\, dy\, dx = \int_{x=0}^{2} \int_{y=0}^{x} e^x (e^y)\, dy\, dx$

$$= \int_{x=0}^{2} e^x \left[\int_{y=0}^{x} e^y\, dy\right] dx$$

$$= \int_{x=0}^{2} e^x \left(e^y\right)_{y=0}^{x} dx = \int_{x=0}^{2} e^x\cdot(e^x-1)\, dx = \int_{x=0}^{2} (e^{2x}-e^x)\, dx$$

$$= \left(\frac{e^{2x}}{2} - e^x\right)_{x=0}^{2} = \frac{e^4}{2} - e^2 - \frac{e^0}{2} + e^0$$

$$= \frac{e^4}{2} - e^2 - \frac{1}{2} + 1 = \frac{e^4}{2} - e^2 + \frac{1}{2} = \frac{1}{2}(e^4 - 2e^2 + 1)$$

$$= \frac{1}{2}(e^2-1)^2.$$

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* Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx\, dy$

Sol:- $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx\, dy = \int_0^\infty \int_0^\infty e^{-y^2} \cdot e^{-x^2} dx\, dy$

$= \int_{y=0}^{\infty} e^{-y^2} \left[\int_{x=0}^{\infty} e^{-x^2} dx\right] dy$

$= \int_{y=0}^{\infty} e^{-y^2} \frac{\sqrt{\pi}}{2} dy \quad \left[\because \int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}\right]$

$= \frac{\sqrt{\pi}}{2} \int_{y=0}^{\infty} e^{-y^2} dy = \frac{\sqrt{\pi}}{2} \cdot \frac{\sqrt{\pi}}{2} = \frac{(\sqrt{\pi})^2}{4} = \frac{\pi}{4}$.

* Evaluate $\int_0^4 \int_0^{x^2} e^{y/x} dy\, dx$

Sol:- $\int_0^4 \int_0^{x^2} e^{y/x} dy\, dx = \int_{x=0}^{4} \left[\int_{y=0}^{x^2} e^{y/x} dy\right] dx$

$= \int_{x=0}^{4} \left[\frac{e^{y/x}}{1/x}\right]_{y=0}^{x^2} dx = \int_{x=0}^{4} \left(x\, e^{y/x}\right)_{y=0}^{x^2} dx$

$= \int_{x=0}^{4} \left[x\, e^{x^2/x} - x\right] dx = \int_{x=0}^{4} \left(x\, e^{x} - x\right) dx$.

$= \left[x e^x - e^x - \frac{x^2}{2}\right]_{x=0}^{4}$

$= 4e^4 - e^4 - \frac{16}{2} - (0-1-0)$

$= 3e^4 - 7$

$\int x e^x dx$ (with $u = x$, $v = e^x$)

$\int uv\, dx = uv_1 - u'v_2 + u''v_3 - u'''v_4 + \cdots$

$\therefore \int x e^x dx = x \cdot e^x - 1 \cdot e^x + 0 \cdot e^x$

$= x e^x - e^x$.

* Evaluate $\int_0^5 \int_0^{x^2} x(x^2+y^2)\, dx\, dy$.

Sol:- $\int_0^5 \int_0^{x^2} x(x^2+y^2)\, dx\, dy = \int_{x=0}^{5} \left[\int_{y=0}^{x^2} (x^3 + x y^2) dy\right] dx$.

$= \int_{x=0}^{5} \left(x^3 y + x \frac{y^3}{3}\right)_{y=0}^{x^2} dx = \int_{x=0}^{5} \left[x^3 \cdot x^2 + x \frac{(x^2)^3}{3} - 0 - 0\right] dx$

$= \int_{x=0}^{5} \left(x^5 + \frac{x^7}{3}\right) dx = \left[\frac{x^6}{6} + \frac{x^8}{3(8)}\right]_0^5$

$= \frac{(5)^6}{6} + \frac{5^8}{24} = \frac{5^6}{24}\left[4 + 25\right] = \frac{29}{24}(5^6)$.

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CHAPTER **1** **Special Functions**

1.1 INTRODUCTION

Special functions are particular mathematical functions occur in mathematical analysis, physics and in many other applications. There is no general formal definition. In particular, elementary functions are also considered as special functions.

Algebraic functions (Polynomial, rational and irrational functions) and Transcendental functions (Trigonometric, Inverse trigonometric, Logarithmic, Exponential and Hyperbolic functions) taken together constitute the **elementary functions.** All functions other than the elementary functions are called **special functions.** These include : Beta, Gamma, Error, Bessel and Legendre functions, Hermite, Laguerre, chebyshev polynomials, sine and exponential integrals, etc.

Many integrals can be evaluated easily by expressing them in terms of Beta and Gamma functions. To find the solution of multi-dimensional heat conduction problems, we require a prior knowledge of Fourier's series, Bessel's functions, Legendre polynomials, Laplace transform methods and complex variable theory.

*1.2 IMPROPER INTEGRALS

[Not included in JNTU(K)]

Consider the integral $\int_a^b f(x)\,dx$. Such an integral, for which

(*i*) either the interval of integration is not finite *i.e.* $a = -\infty$ or $b = \infty$ or both

(*ii*) or the function $f(x)$ is unbounded at one or more points in $[a, b]$ is called an **improper integral.**

Integrals corresponding to (*i*) and (*ii*) are called improper integrals of the first and second kinds respectively. Integrals which satisfy both the conditions (*i*) and (*ii*) are called, improper integrals of the third kind.

Examples :

1. $\int_0^\infty \frac{dx}{1+x^4}$ and $\int_{-\infty}^\infty \frac{dx}{1+x^2}$ are improper integrals of the first kind.

2. $\int_0^1 \frac{dx}{1-x^2}$ is an improper integral of the second kind.

3. The Gamma function defined by the integral $\int_0^\infty e^{-x}x^{n-1}\,dx$ when $n > 0$ is an improper integral of the third kind.

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1.3 BETA FUNCTION

[JNTU Aug. 07S, Nov. 08S (Set No. 1)]
[Not included in JNTU(K)]

Def. The definite integral $\int_0^1 x^{m-1}(1-x)^{n-1}\,dx$ is called the **Beta function** and is denoted by $B(m, n)$ and read as "Beta m, n". The above integral converges for $m > 0$, $n > 0$.

Thus, $B(m,n)=\int_0^1 x^{m-1}(1-x)^{n-1}\,dx$, where $m>0, n>0$. Beta function is also called Eulerian integral of the first kind. This function is closely connected with the Gamma function discussed on page 16.

1.4 PROPERTIES OF BETA FUNCTION

[Not included in JNTU(K)]

(i) Symmetry of Beta function *i.e.*, $B(m, n) = B(n, m)$.

[JNTU Aug. 07S, Nov. 08S (Set No. 1)]

Proof. By definition, we have

$$B(m,n)=\int_0^1 x^{m-1}(1-x)^{n-1}\,dx$$

Put $1 - x = y$ so that $dx = -dy$

$$\therefore B(m,n)=\int_1^0 (1-y)^{m-1}y^{n-1}(-dy)=\int_0^1 y^{n-1}(1-y)^{m-1}\,dy$$

$$=\int_0^1 x^{n-1}(1-x)^{m-1}\,dx=B(n,m)\left[\because \int_a^b f(t)\,dt=\int_a^b f(x)\,dx\right]$$

Hence, $B(m, n) = B(n, m)$

Aliter. We know that $B(m,n)=\int_0^1 x^{m-1}(1-x)^{n-1}\,dx$

From properties of definite integrals, we have

$$\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx$$

$$\therefore \quad B(m,n)=\int_0^1 (1-x)^{m-1}[1-(1-x)]^{n-1}\,dx=\int_0^1 x^{n-1}(1-x)^{m-1}\,dx=B(n,m)$$

(ii) $B(m,n)=2\int_0^{\pi/2}\sin^{2m-1}\theta\,\cos^{2n-1}\theta\,d\theta$ [JNTU 2003, 2005S (Set. No. 2)]

Proof. By definition, we have

$$B(m,n)=\int_0^1 x^{m-1}(1-x)^{n-1}\,dx$$

Put $x=\sin^2\theta$ so that $dx=\sin 2\theta\,d\theta$

$$\therefore \quad B(m,n)=\int_0^{\pi/2}(\sin^2\theta)^{m-1}(1-\sin^2\theta)^{n-1}\sin 2\theta\,d\theta$$

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(11)

$$= \int_0^{\pi/2} \sin^{2m-2}\theta \cos^{2n-2}\theta \,(2\sin\theta \cos\theta)\, d\theta$$

$$= 2\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta \qquad \text{...(1)}$$

Note. From (1), we have

$$\int_0^{\pi/2} \sin^{2m-1}\theta \, \cos^{2n-1}\theta \, d\theta = \frac{1}{2} B(m, n)$$

***(iii)* $B(m, n) = B(m+1, n) + B(m, n+1)$**

Proof. By definition, we have

$$B(m+1, n) + B(m, n+1) = \int_0^1 x^m (1-x)^{n-1} dx + \int_0^1 x^{m-1}(1-x)^n \, dx$$

$$= \int_0^1 \left[x^m (1-x)^{n-1} + x^{m-1}(1-x)^n\right] dx$$

$$= \int_0^1 x^{m-1}(1-x)^{n-1}\left[x + (1-x)\right] dx$$

$$= \int_0^1 x^{m-1}(1-x)^{n-1} dx = B(m, n)$$

Hence, $B(m, n) = B(m+1, n) + B(m, n+1)$.

***(iv)* If m and n are positive integers, then $B(m, n) = \dfrac{(m-1)!\,(n-1)!}{(m+n-1)!}$**

Proof. We have

$$B(m, n) = \int_0^1 x^{m-1}(1-x)^{n-1} dx \qquad \text{[Integration by parts]}$$

$$= \left[x^{m-1}\frac{(1-x)^n}{n(-1)}\right]_0^1 - \int_0^1 \frac{(1-x)^n}{n(-1)}(m-1)\, x^{m-2} dx$$

$$= \frac{m-1}{n}\int_0^1 x^{m-2}(1-x)^n \, dx = \frac{m-1}{n} B(m-1, n+1) \qquad \text{...(1)}$$

Now we have to find $B(m-1, n+1)$. To obtain this put $m = m-1$ and $n = n+1$ in (1). Then, we have

$$B(m-1, n+1) = \frac{m-2}{n+1} B(m-2, n+2)$$

Putting this value of $B(m-1, n+1)$ in (1), we have

$$B(m, n) = \frac{m-1}{n} \cdot \frac{m-2}{n+1} B(m-2, n+2) \qquad \text{...(2)}$$

Changing m to $m-2$ and n to $n+2$, from (1) we have

$$B(m-2, n+2) = \frac{m-3}{n+2} B(m-3, n+3)$$

From (2), we have

$$B(m, n) = \frac{m-1}{n} \cdot \frac{m-2}{n+1} \cdot \frac{m-3}{n+2} B(m-3, n+3) \qquad \text{...(3)}$$

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Proceeding like this, we get

$$B(m,n)=\frac{(m-1)(m-2)(m-3)\dots[m-(m-1)]}{n(n+1)(n+2)\dots[n+(m-2)]}B[m-(m-1),\ n+(m-1)]$$

$$=\frac{(m-1)(m-2)(m-3)\dots 1}{n(n+1)(n+2)\dots(n+m-2)}B(1,\ n+m-1) \qquad \dots(4)$$

But $B(1,n+m-1)=\int_0^1 x^0(1-x)^{n+m-2}dx=\int_0^1(1-x)^{n+m-2}dx$

$$=\left[\frac{(1-x)^{n+m-1}}{(n+m-1)(-1)}\right]_0^1=\frac{-1}{n+m-1}(0-1)=\frac{1}{n+m-1}$$

$\therefore$ From (4), we have

$$B(m,n)=\frac{(m-1)(m-2)(m-3)\dots 1}{n(n+1)(n+2)\dots(n+m-2)}\cdot\frac{1}{n+m-1}$$

$$=\frac{(m-1)!}{(n+m-1)(n+m-2)\dots(n+2)(n+1)n}$$

Multiplying the numerator and denominator by $(n-1)!$, we have

$$B(m,n)=\frac{(m-1)!(n-1)!}{(n+m-1)(n+m-2)\dots(n+2)(n+1)n(n-1)!}=\frac{(m-1)!(n-1)!}{(n+m-1)!}$$

Note : 1. Putting $m=1$, in $B(m,n)=\frac{(m-1)!(n-1)!}{(n+m-1)!}$, we have $B(1,n)=\frac{(n-1)!}{n!}=\frac{1}{n}$

2. Similarly, putting $n=1$, we get $B(m,1)=\frac{1}{m}$

3. The proof of the above result will be easier when we use Gamma function to be introduced later.

1.5 OTHER FORMS OF BETA FUNCTION

Form I. To show [Not included in JNTU(K)]

$$B(m,n)=\int_0^\infty\frac{x^{m-1}}{(1+x)^{m+n}}dx=\int_0^\infty\frac{x^{n-1}}{(1+x)^{m+n}}dx \text{ (or) } \int_0^\infty\frac{y^{n-1}}{(1+y)^{m+n}}dy$$

[JNTU 02S, 03S, 04, 05S, April 2006, Nov. 2008S, JNTU (H) Nov. 2009 (Set No.1)]

(or) $B(p,q)=\int_0^\infty\frac{y^{q-1}}{(1+y)^{p+q}}dy$

[JNTU (K) Nov. 2009 (Set No. 1)]

Proof. We have $B(m,n)=\int_0^1 x^{m-1}(1-x)^{n-1}dx$ ...(1)

Put $x=\frac{1}{1+y}$ so that $dx=-\frac{dy}{(1+y)^2}$

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UNIT-6 **Special Functions.** 113

Improper integrals:- An integral is said to be an improper integral if it satisfying any one of the following conditions

i) either upper limit (or) lower limit (or) both limits are infinite

ii) Integrant of the integral is not bounded in the given interval.

Ex:- $\int_0^3 \frac{1}{(x-1)}\,dx$, $\int_{-1}^{1}\left(\frac{1}{x}\right)dx$

Beta function:- If m,n are positive numbers then the Beta function is denoted by $B(m,n)$ and is defined as

$$B(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx.$$

It is also called as Eulerian Integral of first kind.

Properties of a Beta function:-

i) Symmetric property :- i.e $B(m,n) = B(n,m)$

Proof:- By the definition of Beta function we have

$$B(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$$

put $1-x = y \Rightarrow -dx = dy \Rightarrow dx = -dy$

If $x=0 \Rightarrow y=1$

$x=1 \Rightarrow y=0$

$$= \int_1^0 (1-y)^{m-1}\cdot y^{(n-1)}(-dy)$$

$$= \int_0^1 y^{n-1}(1-y)^{m-1}\,dy. \quad \left[\because \int f(t)\,dt = \int f(x)\,dx = \int f(y)\,dy\right]$$

$$= \int_0^1 x^{n-1}(1-x)^{m-1}\,dx$$

$$\boxed{B(m,n) = B(n,m)}$$

ii) S.T $B(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta\cdot\cos^{2n-1}\theta\,d\theta.$

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Proof :- By definition $B(m,n) = \int_0^1 x^{m-1} (1-x)^{n-1} dx$

put $x = \sin^2\theta$

$dx = 2\sin\theta\cos\theta\, d\theta$

if $x = 0$, $\theta = 0$.

$x = 1$, $\theta = \pi/2$

$= \int_0^{\pi/2} (\sin^2\theta)^{m-1} (1-\sin^2\theta)^{n-1} 2\sin\theta\cos\theta\, d\theta$

$= \int_0^{\pi/2} 2\sin^{2m-2}\theta \cdot \cos^{2n-2}\theta \cdot \sin\theta \cdot \cos\theta\, d\theta$

$= 2\int_0^{\pi/2} \sin^{2m-1}\theta \cdot \cos^{2n-1}\theta\, d\theta$

$$\boxed{B(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta \cdot \cos^{2n-1}\theta\, d\theta}$$

(iii) $B(m,n) = B(m+1,n) + B(m,n+1)$

Proof :- By def $B(m,n) = \int_0^1 x^{m-1} (1-x)^{n-1} dx$

$B(m+1,n) = \int_0^1 x^{m+1-1} (1-x)^{n-1} dx$

$= \int_0^1 x^m (1-x)^{n-1} dx$

$B(m,n+1) = \int_0^1 x^{m-1} (1-x)^{n+1-1} dx$

$= \int_0^1 x^{m-1} (1-x)^n dx$

By adding $B(m+1,n) + B(m,n+1) = \int_0^1 x^m (1-x)^{n-1} dx + \int_0^1 x^{m-1} (1-x)^n dx$

$= \int_0^1 x^{m-1} (1-x)^{n-1} [x + (1-x)] dx$

$= \int_0^1 x^{m-1} (1-x)^{n-1} (1) dx$

$= B(m,n)$

$$\boxed{B(m+1,n) + B(m,n+1) = B(m,n)}$$

Other forms of Beta function :-

Q. P.T $B(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx = \int_0^\infty \frac{x^{n-1}}{(1+x)^{m+n}} dx$.

Proof:- By def $B(m,n) = \int_0^1 x^{m-1} (1-x)^{n-1} dx$ —① .

put $x = \frac{1}{1+y} \Rightarrow dx = \frac{-1}{(1+y)^2} dy$

If $x = 0 \Rightarrow y = \infty$

$x = 1 \Rightarrow y = 0$

① becomes

$$B(m,n) = \int_\infty^0 \left(\frac{1}{1+y}\right)^{m-1} \left[1 - \frac{1}{1+y}\right]^{n-1} \cdot \frac{-1}{(1+y)^2} dy$$

$$= \int_0^\infty \frac{1}{(1+y)^{m-1}} \left[\frac{1+y-1}{1+y}\right]^{n-1} \frac{1}{(1+y)^2} dy$$

$$= \int_0^\infty \frac{1}{(1+y)^{m-1}} \left(\frac{y}{1+y}\right)^{n-1} \frac{1}{(1+y)^2} dy$$

$$= \int_0^\infty \frac{y^{n-1}}{(1+y)^{m-1+n-1+2}} dy$$

$$= \int_0^\infty \frac{y^{n-1}}{(1+y)^{m+n}} dy = \int_0^\infty \frac{x^{n-1}}{(1+x)^{m+n}} dx$$

$$\boxed{B(m,n) = \int_0^\infty \frac{x^{n-1}}{(1+x)^{m+n}} dx}$$

Since it satisfies symmetric property i.e $B(m,n) = B(n,m)$

Hence $B(m,n) = \int_0^\infty \frac{x^{n-1}}{(1+x)^{m+n}} dx = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx$

Q. P.T $B(m,n) = \int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} dx$.

Proof:- By using the other form of the Beta functions we have $B(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx$

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$$= \int_0^1 \frac{x^{m-1}}{(1+x)^{m+n}}\,dx + \int_1^{\infty} \frac{x^{m-1}}{(1+x)^{m+n}}\,dx \quad \text{—— (1)}$$

To evaluate the given Beta function put $x = \frac{1}{y}$ in second part of integral we get

put $x = \frac{1}{y} \Rightarrow dx = \frac{-1}{y^2}\,dy$

If $x = 1, \Rightarrow y = 1$

$x = \infty \Rightarrow y = 0$

Now second part of integral becomes

$$\int_1^0 \frac{\left(\frac{1}{y}\right)^{m-1}}{\left(1+\frac{1}{y}\right)^{m+n}} \frac{-1}{y^2}\,dy = \int_1^0 \frac{\frac{1}{y^{m-1}}}{\left(\frac{1+y}{y}\right)^{m+n}} \cdot \frac{-1}{y^2}\,dy$$

$$= \int_0^1 \frac{1}{y^{m-1}} \times \frac{y^{m+n}}{(1+y)^{m+n}} \cdot \frac{1}{y^2}\,dy$$

$$= \int_0^1 \frac{y^{m+n-m+1-2}}{(1+y)^{m+n}}\,dy = \int_0^1 \frac{y^{n-1}}{(1+y)^{m+n}}\,dy = \int_0^1 \frac{x^{n-1}}{(1+x)^{m+n}}\,dx$$

From (1) $B(m,n) = \int_0^1 \frac{x^{m-1}}{(1+x)^{m+n}}\,dx + \int_1^{\infty} \frac{x^{m-1}}{(1+x)^{m+n}}\,dx$

$$B(m,n) = \int_0^1 \frac{x^{m-1}}{(1+x)^{m+n}}\,dx + \int_0^1 \frac{x^{n-1}}{(1+x)^{m+n}}\,dx$$

$$\boxed{B(m,n) = \int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}}\,dx}$$

8) S.T $\int_0^{\infty} \frac{x^{m-1} - x^{n-1}}{(1+x)^{m+n}}\,dx = 0$ & $\int_0^{\infty} \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}}\,dx = 2B(m,n)$

Proof:- We know that $\int_0^{\infty} \frac{x^{m-1}}{(1+x)^{m+n}}\,dx = \int_0^{\infty} \frac{x^{n-1}}{(1+x)^{m+n}}\,dx = B(m,n)$

$$\int_0^{\infty} \frac{x^{m-1} - x^{n-1}}{(1+x)^{m+n}}\,dx = \int_0^{\infty} \frac{x^{m-1}}{(1+x)^{m+n}}\,dx - \int_0^{\infty} \frac{x^{n-1}}{(1+x)^{m+n}}\,dx$$

$$= B(m,n) - B(m,n) = 0.$$

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Result Analysis

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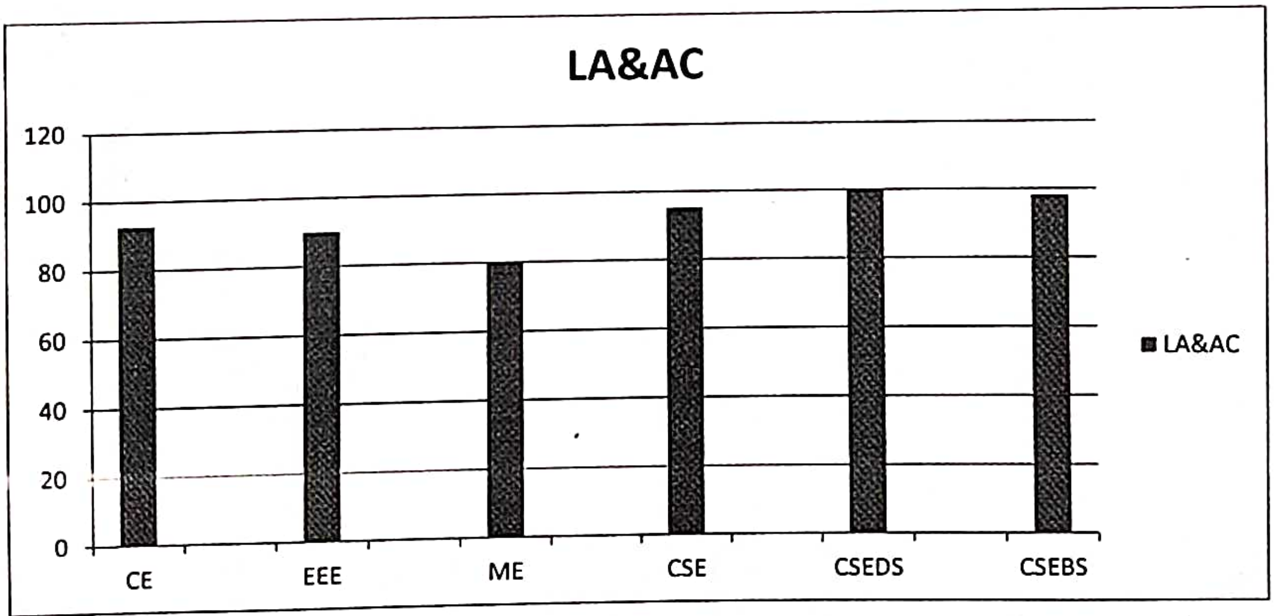
RESULT ANALYSIS

Academic Year: 2020-21, I- I Sem.

LINEAR ALGEBRA AND ADVANCED CALCULUS (LA&AC)

S.No	Branch	Number of Appeared	Number of Passed	Number of failed	Pass %
1	CE	96	89	07	92.71
2	EEE	100	90	10	90
3	ME	86	69	17	80.23
4	CSE	191	182	09	95.29
5	CSEDS	64	64	00	100
6	CSEBS	55	54	01	98.18



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