Shear Centre

13.1 Introduction

The shear centre is the point through which if the resultant shear force acts then member is subjected to simple bending without twisting. It means a load acting on a beam through shear centre will produce bending without torsion or twisting. Shear centre is also called **centre of flexure**.

13.2 Location of Shear Centre

- Shear centre generally does not coincide with the centroid of section except in special cases when the area is symmetrical about both axis.
- Shear centre always lies on the axis of symmetry if exists.

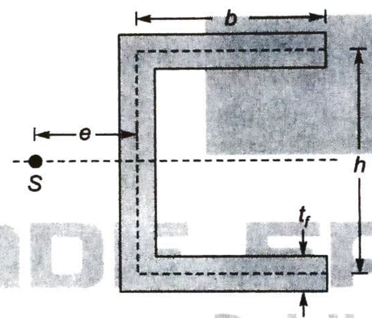


Fig. 13.1

- If there are two or more than two axis of symmetry exist, then shear centre will coincide with point of intersection of axis of symmetry. In this case shear centre of area will be same as centroid of area.

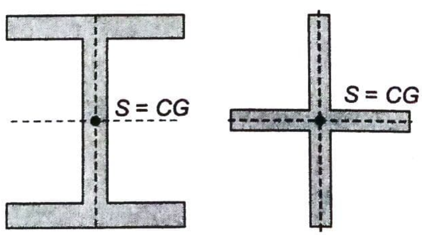


Fig. 13.2

where, S = shear centre and CG = centroid

- If a section is made of two narrow rectangles then shear centre lies on the junction of both rectangles

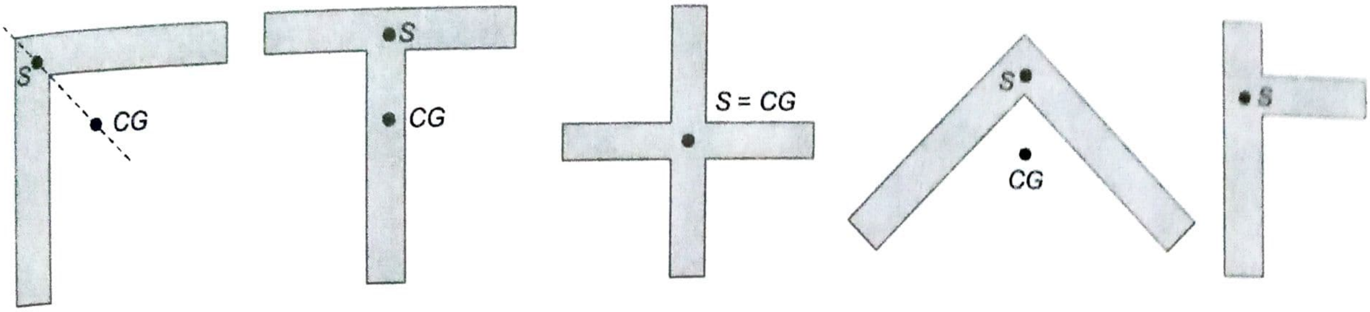


Fig. 13.3

> **NOTE**
>
> - Beams with thin walled open cross-section like channels, angles, T-beams etc. are torsionally very weak. Therefore, it is important to locate the shear centre (S) and to take into account the effect of twisting.
> - For solid sections and closed hollow sections, the shear centre (S) is located near the centroid. Such sections have high torsional rigidity, hence if the load passes near the centroid then torsion may be disregarded.

13.3 Shear Flow

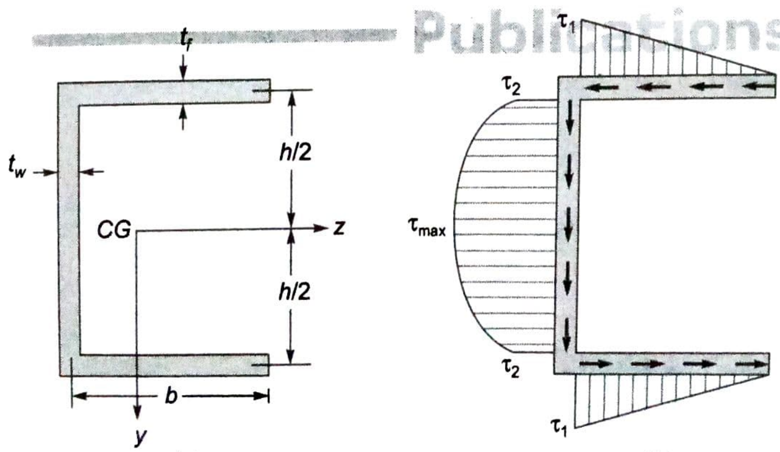
The variation of shear free per unit length is known as shear flow

$$\text{Longitudinal shear flow} = \frac{SA\bar{y}}{I}$$

where, S = shear force

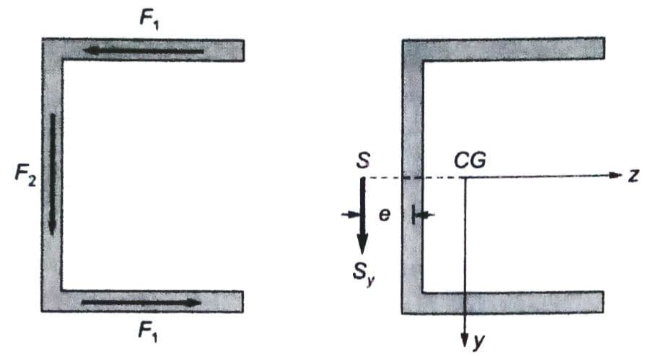
$A\bar{y}$ = moment of area under consideration about NA

I = moment of inertia about neutral axis



13.4 Shear Centres of Thin-walled Open Sections

1. **Channel section:** Consider a channel section as shown in figure 13.5, bending take place about z-axis. Assuming section is subjected to vertical shear force S_y parallel to y-axis.



(a) (b)

Fig. 13.5

We know shear stress in beams of thin walled open cross-section is given by,

$$\tau = \frac{S_y A\bar{y}}{It}$$

Where, S_y = vertical shear force parallel to y-axis

Hence, maximum shear stress in flange τ_1 is given as

$$\tau_1 = \frac{S_y A_f \bar{y}}{I_z t_f}$$

where, $$A_f \bar{y} = \frac{b t_f h}{2}$$

$$\therefore \quad \tau_1 = \frac{S_y b h}{2I_z} \quad \text{...(i)}$$

Similarly, τ_2 (just within the web at junction) = $\dfrac{S_y b t_f h}{2I_z t_w}$ ...(ii)

Also, at the neutral axis shear stresses will be maximum and is given by

$$\tau_{max} = \left(\frac{bt_f}{t_w} + \frac{h}{4}\right)\frac{hS_y}{2I_Z} \quad \text{...(iii)}$$

The total shear force in flange either top or bottom is given as

$$F_1 = \left(\frac{1}{2} \times b \times t_f\right) \times \tau_1 = \frac{hb^2 t_f S_y}{4I_Z}$$

The vertical shear force F_2 in web will be equal to S_y

$$F_2 = (ht_w)\tau_2 + \frac{2}{3}(\tau_{max} - \tau_2)ht_w$$

From eq. (ii) and (iii) we get,

$$F_2 = \left(\frac{t_w h^3}{12} + \frac{bh^2 t_f}{2}\right)\frac{S_y}{I_Z}$$

where, $$I_z = \frac{t_w h^3}{12} + \frac{bh^2 t_f}{2}$$

Let shear centre is at a distance 'e' from the centre line of the web of section, then as per the definition of shear centre

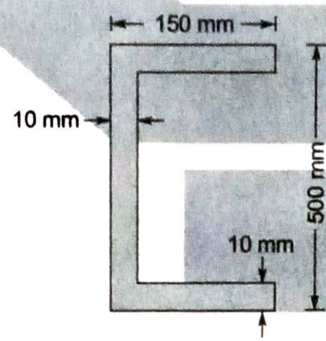
$$F_1 \times h - F_2 \times e = S_y \times 0$$

$$\Rightarrow \left(\frac{hb^2 t_f S_y}{4I_z}\right) \times h - \left(\frac{t_w h^3}{12} + \frac{bh^2 t_f}{2}\right)\frac{S_y}{I_z} \times e = 0$$

$$\therefore \quad \frac{b^2 h^2 t_f}{4} - I_z \times e = 0$$

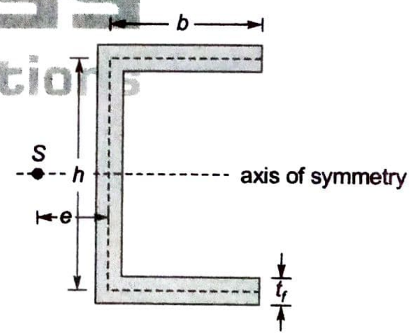
$$e = \frac{b^2 h^2 t_f}{4I_z}$$

Example 13.1 **Locate the position of shear centre for a thin channel section having total depth of 500 mm, width of flange 150 mm and thickness of web and flange is 10 mm each.**



Solution:

We know shear centre (S) will lie on axis of symmetry. Let shear centre (S) is at a distance e from the centre line of web.



$$e = \frac{h^2 b^2 t_f}{4I_z}$$

where, $h = 500 - 5 - 5 = 490$ mm

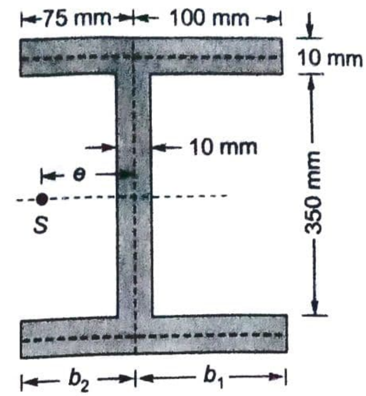
$b = 150 - 5 = 145$ mm

$t_f = 10$ mm

$$I_z = I_{x1} - I_{x2} = \frac{150 \times 500^3}{12} - \left[\frac{(150-10)\times(500-20)^3}{12}\right] = 2722.6 \times 10^5 \text{ mm}^4$$

Hence, $$e = \frac{490^2 \times 145^2 \times 10}{4 \times 2722.6 \times 10^5} = 46.35 \text{ mm}$$

Example 13.2 For given unsymmetrical I-section. Locate position of shear centre.



Solution:

$$b_2 = 75 \text{ mm}$$
$$b_1 = 100 \text{ mm}$$
$$t_f = 10 \text{ mm}$$
$$h = 360 \text{ mm}$$

Position of shear centre,

$$e = \frac{h^2 t_f \left(b_1^2 - b_2^2\right)}{4I_z}$$

$$I_z = I_{web} + 2I_{Flange} = \frac{10\times 350^3}{12} + 2\left[\frac{175\times 10^3}{12} + 175\times 10\left(\frac{350}{2}+5\right)^2\right]$$

$$= 35729166.67 + 113429166.7 = 1.492\times 10^8 \text{ mm}^4$$

$$\therefore \quad e = \frac{360^2\times 10\times\left(100^2 - 75^2\right)}{4\times 1.492\times 10^8} = 9.50 \text{ mm}$$

13.5 Thin Walled Semicircular Cross-section

Shear stress at an angle θ is given by

$$\tau = \frac{S}{It}\int y\,dA \quad \text{...(i)}$$

where $\int y\,dA$ = moment of area under consideration about NA

$$= \int (R\cos\theta)(Rd\theta\times t) = R^2 t \sin\theta$$

I = MOI of entire section about NA

$$= \int y^2 dA = \int_0^{\pi} (R\cos\theta)^2 tRd\theta$$

$$= R^3 t\int_0^{\pi}\cos^2\theta\,d\theta = \frac{\pi R^3 t}{2}$$

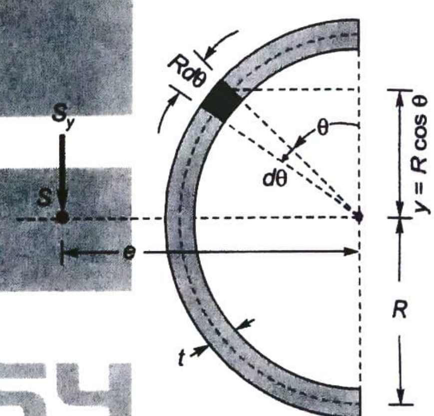


Fig. 13.6

Now from (i)

$$\tau = \frac{S\times R^2 t\sin\theta}{\left(\frac{\pi R^3 t}{2}\right)\times t} = \frac{2S}{\pi Rt}\sin\theta \quad \text{...(ii)}$$

According to the definition of shear centre, section will be free from any torsion (twisting). Then moment due to shearing stress about centre 'O' must be equal to moment due to resultant shear S_y.

Hence, Moment due to shearing stress = Moment due to S_y

$$\Rightarrow \quad (\tau dA)\times R = S\times e$$

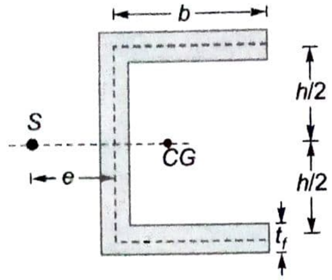
$$\Rightarrow \quad \int_0^{\pi}\left(\frac{2S\sin\theta}{\pi Rt}\right)tRd\theta\times R = S\times e$$

$$\Rightarrow \quad \frac{4RS}{\pi} = S\times e$$

$$\therefore \quad e = \frac{4R}{\pi}$$

13.6 Shear Centres of Some Important Sections

(a) Channel Section

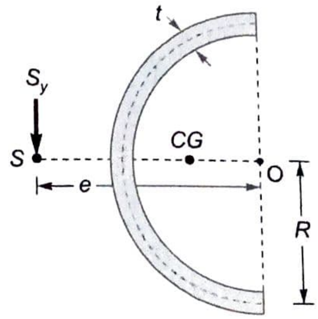


$$e = \frac{b^2 h^2 t_f}{4I_z}$$

where, I_Z = moment of inertia about axis of symmetry.

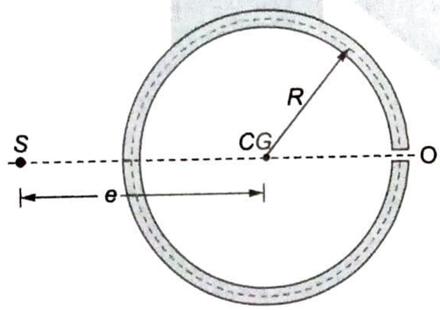
t_f = thickness of flange

(b) Semi Circular Section



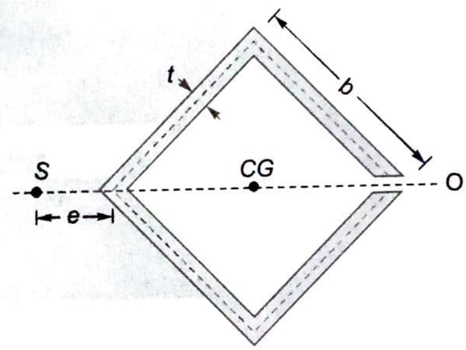
$$e = \frac{4R}{\pi} = 1.27R$$

(c) Slit Circular Tube with Constant Thickness



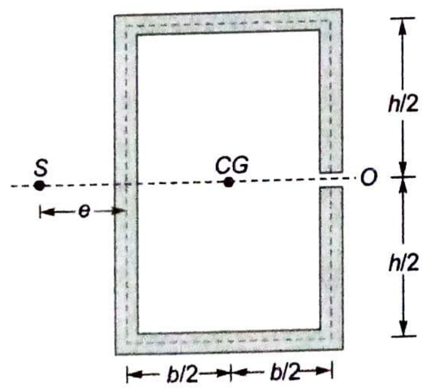
$$e = 2R$$

(d) Slit Square Tube



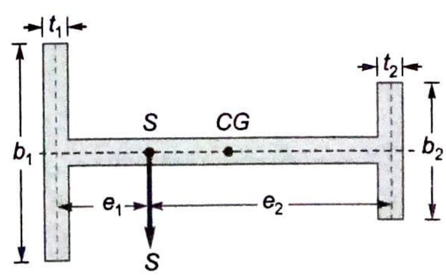
$$e = \frac{b}{2\sqrt{2}}$$

(e) Rectangular Tube of Constant Thickness having Slit at End



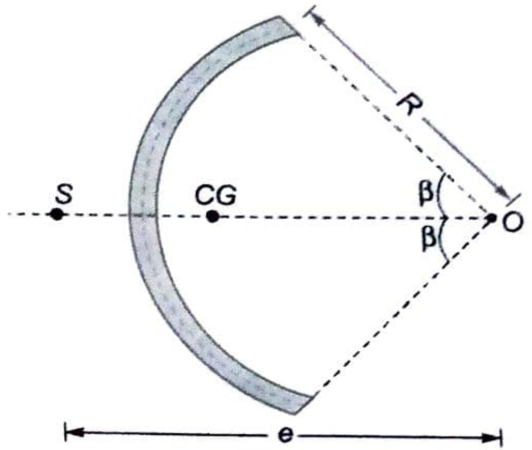
$$e = \frac{b(2h+3b)}{2(h+3b)}$$

(f) I-section (Symmetric about One Axis Only)



$$e_1 = \frac{t_2 b_2^3 h}{t_1 b_1^3 + t_2 b_2^3}\ ;\ e_2 = \frac{t_1 b_1^3 h}{t_1 b_1^3 + t_2 b_2^3}$$

(g) Circular Arc



$$e = \frac{2R(\sin\beta - \beta\cos\beta)}{(\beta - \sin\beta\cos\beta)}$$